Introductory Note

The National Productivity Council, since 1958, has conducted a large number of training programmes in the fields of Industrial Management, Industrial Relations, and Industrial Engineering. Its specialists have also been carrying out an increasing number of consultancy assignments in a wide variety of organisations using different tools and techniques of productivity improvement. These experiences are being consolidated in the form of Training Manuals, as they would be, of benefit not only to its own specialists, but also to the Local Productivity Councils, and other training organisations in the country.

The Training Manuals, are just an attempt at systematising and consolidating the course contents, which would undergo progressive improvement from time to time. It is the experience of the Council that a good Training Manual in the field of productivity should be essentially practice-oriented, illustrating the applied aspects for effective communication to the trainees. It is hoped that the experience of the NPC specialists as trainers-cum-consultants would make the Training Manuals useful to the needs of Indian industries and other organisations.

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History and Development

1. Introduction

1.1 Management today is faced with unprecedented and complex problems. The rapidly changing technology, the increasing complexity of a company's operations, and an intensified competitive environment-all have combined to make management's problems more difficult. The time available for decision-making g is shortened and the adverse effects associated with incorrect decision have increased. For Long, the poorly designed planning and control methods failed to cope up with the changing realities of modern business life and consequently the management has continuously been seeking techniques to make planning and control more effective, particularly when a complex set of activities, functions and inter-relations are involved. To meet this unusual need for better decision-making tools, the past decade has witnessed a steady growth in the application of analytical aids to planning. In this area the development and application of PERT-Programme Evaluation and Review Technique-has been perhaps the most significant.

1.2 Certainly PERT is not an altogether new technique. It is partially evolutionary and partially new creation. Today, most functioning systems are hydrid. They always take on the desirable characteristics of the other and the more they do so the more the differences that existed initially tend to disappear. One thing should be clear that PERT is not, and is not intended to be, a production control tool. It is primarily designed for use in complex projects in which it is difficult to keep track of progress in all areas. It provides the timely information so that immediate evaluation and review of the progress made so far is possible and could be followed -by corrective action.

2. Historical Development

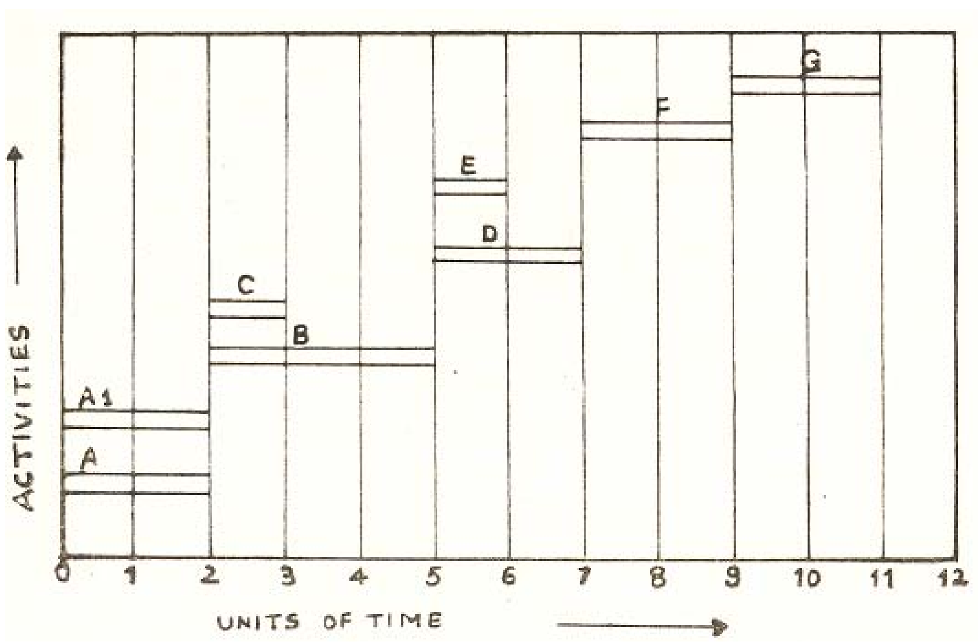
2.1 In earlier times certain pictorial depictions were used for this purpose. One of these which became quite popular was known as 'Bar Chart' developed by

Gantt around 1900. It consisted of two axes showing time and the other indicating jobs performed.

The Job/Activities are represented by bars and lengths of bars indicate the duration of the job/activity completion. This would be clear from the following illustration:

Activity	Depends on	Duration
A	–	2
B	A	3
C	A	1
D	B,A,	2
E	B,C,A_I	1
F	D,E	2
G	F	2
Al	–	2

The Above example can be shown by bar chart as follows:



With the help of the above bar chart it is easy to construct and provide a visual representation of the project which shows exactly when each of the above activities are supposed to start and finish. Though it gives an excellent pictorial depiction but it does not bring out :

(1) Sequence and inter dependies of the various activities, which is of very vital importance for the project. From the above chart it appears that activity B & C start after the completion of both the activities A & Al but it is not so. Activity 'B' depends on activity 'A' and similarity 'c' depends on activity 'A' only. Hence bar chart has given rise to superfluous relationships of 'B' and 'c' with 'AI"

(2) Its difficult to visualise critical path & floats.

(3) The monitoring and control of the project is difficult.

The above mentioned weaknesses and deficiencies were overcome in subsequent years by evolution of PERT/CPM as one of the important planning and scheduling techniques.

2.2 Both PERT and CPM (Critical Path Method) arrived on the industrial scene at about the same time and as essentially independent developments. In 1958, the U.S. Navy Special Projects Office recognised the need for an integrated management control technique for use in their Polaris missile programme. Some 2,000 contractors were involved in the development of the Polaris weapon system. Their diverse activities had to be co-ordinated closely to ensure timely and successful completion of the Polaris. PERT was developed in response to this need. Its dramatic success in bringing the project to completion years ahead of schedule provided impetus to its wide acceptance as a management tool in defence and commercial organisations of United States.

2.3 Concurrently, the management control technique known as Critical Path Method (CPM) was being developed by the Engineering Services Divisions of Du Pent and Remington Rand, U.S.A. to meet similar management needs of private industries. While superficially different, both PERT and CPM proceeded from the basic concept of using a network as a model for an actual project. Those differences that existed initially have tended to disappear as each system took on desirable characteristics of the other. Today, most functioning systems in industry are hybrid.

2.4 Because of the similarity between PERT and CPM, as used currently, there is little to gain by differentiating between the two. This training kit will treat them as subsets of a single control system based on application of the network technique. For the sake of simplicity, this system will be referred to as PERT.

3. Project Management

3.1 PERT is essentially used for project management which is basically of no repetitive nature. A project, as defined here, is usually, but not always a one-time effort. Although similar work may have been done previously, it is not being repeated in the identical manner on a production basis. Consequently, in order to accomplish the project tasks efficiently the project manager must plan and schedule largely on the basis of his experience: with similar projects, applying his judgment to the particular conditions of the project at hand. During the course of the project, he must continually replant and reschedule because of unexpected progress, delays, or other changes in technical conditions.

3.2 Until just a few years ago, there was no generally accepted formal procedure to aid management of projects. Each manager had his own scheme, which often involved limited use of bar charts-a useful tool in production management but inadequate for the complex inter-relationships associated with project management. The development of critical path methods in 1958·59 had provided the basis for a more formal and general approach toward a discipline of project management. Critical path methods involve a graphical portrayal of the inter-relationships among the elements of a project and an arithmetic procedure which identifies the relative importance of each element in the over-all schedule. Since their development, critical path methods have been applied with notable successes to large-scale research and development programmes, construction work, industrial maintenance and installation operations, and even the production of motion pictures and the planning of open heart surgery; according to our definition, all of these are classed as projects. Applications have also included production planning, where introduction of a new product or changeover to a new model is essentially a project.

3.3 In all of these projects one is, to a greater or lesser degree, concerned with developing an optimal (or at least a workable) plan of activities that make up lie project, including a specification of their interrelationships. Also, one is interested in scheduling these activities in an acceptable time span, and finally with "controlling" the conduct of the scheduled work.

3.4 With respect to planning and scheduling, one must consider the manpower and the facilities required for carrying out the programme as it progresses in time. The aim is to plan the conduct of the programme so that the cost and time required to complete the project are properly balanced, and excessive demands of key resources are avoided. With respect to the control function, one is concerned with moldering the expenditure of time and money in carrying out the scheduled programme, as well as the resulting "product" quality or performance. For the most part, critical path methods have concentrated on the time parameter and to a somewhat lesser extent on the cost parameter.

4. Basic Steps

4.1. A summary of the steps involved in applying PERT is given below:

Step 1: Project Planning. The activities making up the project are defined and their dependency upon one another is shown explicitly in the form of a network diagram.

Step 2: Time Estimation. Estimates of the time required to perform each of the network activities are made; these estimates are based upon manpower and equipment availability and certain assumptions that may have been made in planning the project in Step 1.

Step 3: Scheduling. The Scheduling computations give the earliest and the Latest allowable start and finish times for each activity, and as a byproduct, they identify the critical path through the network, and indicate the amount of "slack" time associated with the non-critical paths.

Step 4: Time-cost Trade-offs. If the scheduled time to complete the project as determined in Step 3 is satisfactory, the project planning and scheduling may be complete. However, if one is interested in determining the cost of reducing the project completion time, then time-cost trade-offs of activity performance times must be considered for those activities, on the critical and near-critical path(s).

Step 5: Resource Allocation. The feasibility of each schedule must be checked with respect to manpower and equipment requirements. Establishing complete feasibility of a specific schedule may require replanning and rescheduling (Steps 1 and 3) or time-cost trade offs (Step 4). He r ce, a final solution may require performance of a number of cycles of steps 3, 4 and 5.

Steps 6: Project Control. When the network plan and the schedule have been developed to a satisfactory extent, they are prepared to final form for use in the field. The project is controlled by checking progress against the schedule, assigning and scheduling manpower and equipment, and analysing the effects of delays. Whenever major changes are made in the schedule, the network is revised accordingly and a new schedule is computed.

Promise For The Future

5.1 Certainly PERT or, more correctly, the network analysis technique has opened the door to a better understanding of management system. The network theory is awakening managers to the many inter-related complexities of business operations. A management system is composed of equipment, skills, procedures, and techniques-the composite of which forms an instrument of administration or control. Networks developed today for programmes are now beginning to include a greater number of administrative actions, paperwork, and decision points. Eventually, managers will be able to plot on networks most organisational functions that may be planned in the future, including such routine matters as the setting up of financial targets, scheduling of recurring reports and meetings, and labour contract negotiations.

5.2 By using automated control information, management will be alerted when even these routine functions arc not being carried out according to plan. In other words, network techniques will provide a true means for identifying exceptions so that we can manage by exception. Many supervisors claim that they manage by exception, but they have no way of identifying the exceptions. Undoubtedly, the increased awareness of management systems and the increased use of network theory in general administration will be one of the most significant outgrowths of the present PERT application .

Developing the Project Network

1. Basic Terminology

1.1 Planning means thinking ahead, based on facts and figures and an intimate knowledge of the task in hand. Planning through PERT is no different. A thorough knowledge of the project in hand is a pre-requisite for PERT. The first step in utilising PERT is the identification of all the activities involved in the project and a graphical representation of their inter-relationship. In the application of PERT / CPM most of the efforts are generally directed towards the development of project network. In this "Planning phase", PERT uses certain definite terminology with specific meanings.

(i) **An Activity:** An Activity represents efforts applied over a period of time which has a definite beginning and ending. This involves expenditure of resources. Activities are graphically represented by arrows with descriptions and time estimates written along the arrow as shown below:

Paint Wall →
10

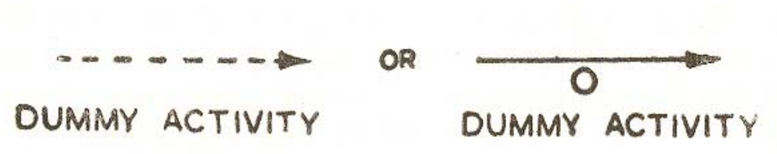
The arrow representing an activity is not a vector quantity. The direction of the arrow and its length has got no specific meaning.

(ii) **An Event:** An Event is a point of time indicating the start or completion of an activity and does not signify any expenditure of resources. An activity is bounded by two events. Events are graphically represented by circles.

Events are described by such words as complete, start, issue, approve, tested, etc. Words like design, procure, test, develop, prepare, etc. indicate that work is being accomplished and thus represent activities.

Events are as such the beginning and the ending points of activities. Some times an event may represent the joint completion of more than one activity in which case it is called a "merge" event. In case of joint starting of more than one activity, the corresponding event is called a "burst" event. The event numbers are written inside as represented below :

(iii) **A Dummy Activity:** Sometimes an arrow is to be used merely to represent dependency of one activity over another whereas the arrow itself does neither represent any expenditure of resources nor it has got any duration. Dummies are generally represented by dotted arrows or solid arrows with zero time duration. These are required for completing the logic of the project.



(iv) **Interdependency:** A project is made up of many activities. These activities are inter-related. From the knowledge of the project in hand the first step is to break down the whole project into a number of smaller activities and find out their interdependency relationship. We examine each activity in relation to other activity and ask ourselves the following questions:

(a) What other activity (ies) must be completed before this activity can start? (Precedence).

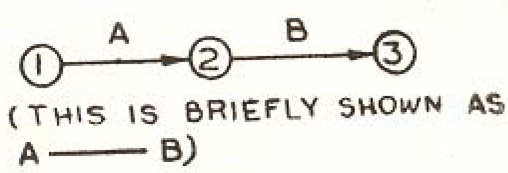
(b) What other activity (ies) can be done while this activity is being done? (Concurrence).

(c) What activity (ies) cannot start until after this activity is done? (Subsequence).

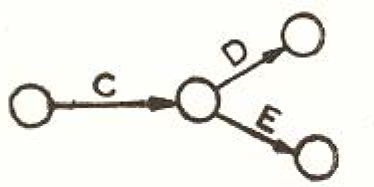
It is to be noted that concurrence is only a relationship of logic, not of time. Duration of each activity does not come into the picture at this phase, only the logical sequence of the individual activities does.

Some examples of the interdependency are illustrated with interpretation:

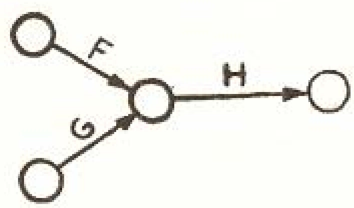
Representation of Interdependency | *Interpretation*



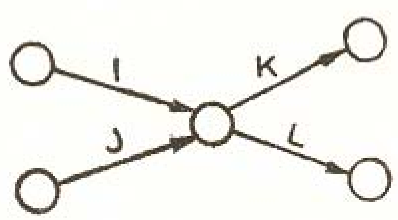
Activity B cannot start until activity A is completed This is in said B depends on A, or B is preceded by A, or A is succeeded by B is succeeded by B. Activity A is also said to be Activity 1-2, i.e., by the event nos.



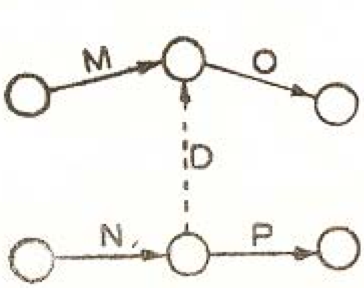
C must be completed before either D or E can start.



H cannot start until both F and G are completed. Which of the two precedent activities F and G will finish first is a matter of schedule and will be determined later. This does not imply that F and G end simultaneously.



Both I and J must be finished before either K or L can start. There is no implication that both I and J will end simultaneously or K and L start at the same time.



o is dependent on both M and N; but P is dependent on only Nand not M. 0 is called the dummy activity and acts to correct the logical sequence.

The logic always flows from tailhead sequence of arrows including that of a dummy. Most activities will flow from left to right, but any individual arrow may go up or down dependency relationship flowing along the path of the arrow in the tail to head sequence.

(v) **Network:** A network is a, graphical representation of the interdependency of all the activities of a project. Networks are also called Arrow Diagrams.

2. Network Development-Visual Method

2.1 The first step in this method of network development is the listing of all activities of the project and preparing a list of interdependency of activities pair by pair. The normal way to start a network is to draw an arrow for the first activity. Add to that its succeeding activity (ies) and so on tilt the final activity is reached. In proceeding this way one will generally come across more than one parallel line (paths), all of which must be similarly followed and completed till the end event.

2.2 Another way is to start with the last activity working backwards, thinking of the activities that must be completed in sequence. As a matter of fact an experienced network diagrammer can start with any activity and can proceed forward or backward representing the interdependency as listed in sequence till an the relationships are covered.

2.3 The first draft of a network (except the very simple one) is always clumsy which has to be changed frequently to give it a proper look. One point that should be kept in mind is the need to minimise the use of dummies, keeping in tact the network logic.

2.4 The following example is a simplified list of activities for a' project to plan and conduct a market survey:

(i) Study the purpose of the survey (A).

(ii) After the study, data collection personnel are hired (B).

(iii) Simultaneously survey questionnaire is designed (C).

(iv) The personnel are then trained in the use of the questionnaire (D).

(v) After designing the questionnaire, the design group can select the household to be surveyed(E).

(vi) The survey is then accomplished and results analysed (F) .

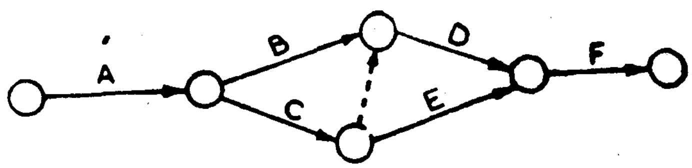
2.4.1 From a scrutiny of the above list of activities the dependency relationship is charted below:

A → B
A → C
B → D
C → D
C → E

C → E
E → F

DEPENDENCY RELATIONSHIP CHART

2.4.2 Proceeding with the initial activity A and charting gradually the above relationships the completed network is as follows:



COMPLETED NETWORK

2.4.3 It is obvious that to keep the logic and dependency intact insertion of one dummy is necessary.

2.5 Network Logics

2.5.1. Before an activity may begin all activities preceding it must be completed.

2.5.2 Length, Orientation or shape of the arrow has no significance. It only implies logic.

2.5.3 The beginning of the activity is a 'tail event' while completion is 'head event'.

2.5.4 No activity to be shown more than once in network.

2.5.5 Any two events may be directly connected by not more than one activity.

2.5.6 While drawing network it is assumed that time flows from left to right and head event number is higher than the tail event number.

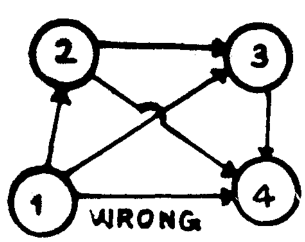
2.5.7 Network may have only one initial event and only one terminal event.

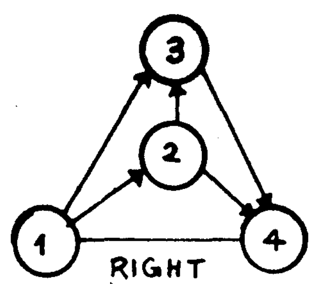
2.5.8 Representation of inter dependency of the activity should be clearly indicated.

2.6. Hints for drawing network.

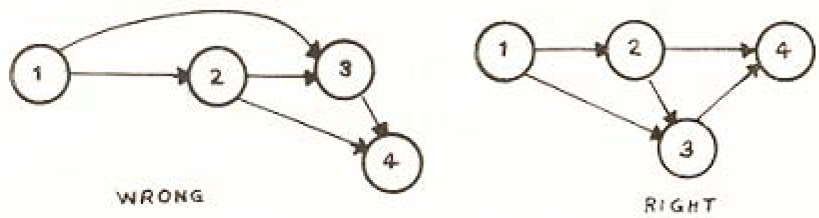
2.6.1 Many of the rules for drawing network are based upon common approach. The few illustrations given below will highlight these aspects.

2.6.1.1 Avoidance of crossing of activities

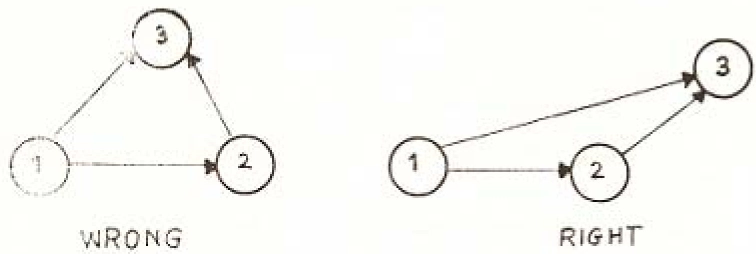




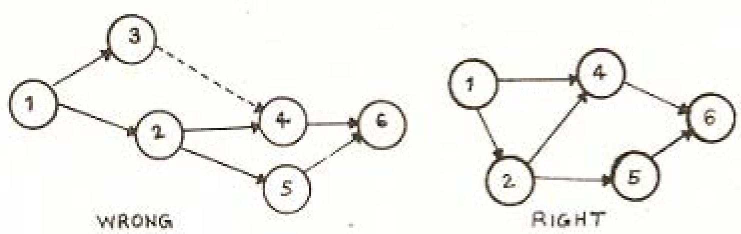
2.6.1.2 Avoidance of loop



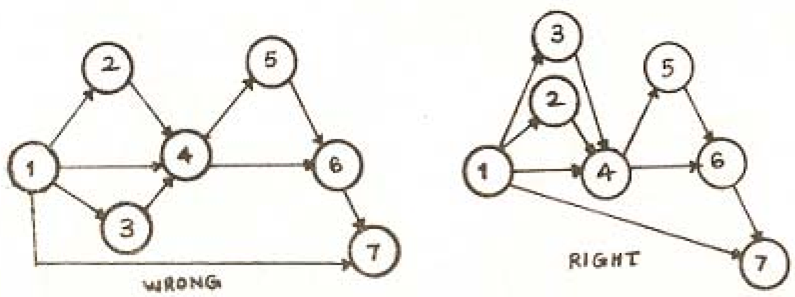
2.6.1.3 Unidirectional representation i.e. left to right.



2.6.1.4 Avoidance of unnecessary dummies



2.6.1.5 Rearranging of arrows length and angle



3. Organisation and Level of Details

3.1 Whether a certain project network will have 200 activities or 2000 activities depends upon the degree of detail desired by the management. The rule to follow is that an activity should represent the smallest unit over which control is desired. There are many other factors involved in determining the most appropriate level. Some of the factors are :

(i) The interests and the span of control of the person using the network.

(ii) Feasibility of expanding the activity into more details.

(iii) Existence of separate skills, facilities and areas of responsibility in the activity.

(iv) Accuracy of the logic and time estimates.

3.2 These are only guidelines., The general approach should be that the network should not be too condensed so as to have no real meaning; neither It should be too elaborate in details making it too expensive and perhaps impractical to control.

3.3 The basic organisational approach in controlling a project by network is by phY3ical subdivisions of the project e.g. spans of bridge, the sub-assemblies of a product, etc. The physical grouping of work in the network should be in relation to the responsibility groups of the management wherever possible. This, however, is not always possible, e.g., the electrical work in a project of building a house cannot appear as a separate well-knit portion of the network, but will appear throughout in relation to other physical activities in their true dependency relationship.

EXERCISE

1. Chart the following:

(a) Procure equipment; inspect equipment.

(b) Hire operator; train operator.

(c) Combine (a) and (b) .

(d) Add "Prepare Inspection Procedure" to (c).

2. Chart the following:- (The arrow denotes 'Constraint on': e.g., A ⟶ B

A⟶C;	B⟶C;	A⟶H;	B⟶H;
C⟶D;	C⟶G;	C⟶F;	D⟶E;
F⟶I;	G⟶H;	H⟶I;	E⟶I;

3. Based on Memorandum of the project 'INDO-EASY' (PERT 21) develop the actual interdependency of the activities.

   Using the inter-relationships develop the project network by visual method.

4. Based on data of the project 'RAYSEE' (PERT 21) draw an inter-relationship chart of all activities and develop the network using the interdependency chart.

Basic Scheduling Computations (PERT TIME)

1. Time Estimates

1.1 After construction of a network, the next step is to relate the important characteristics of the activities it contains. The most important attribute of an activity is the length of time. it will take to accomplish it. This is called the activity duration which means the elapsed time of the activity expressed in working days, hours or weeks without including uncontrollable contingencies such as fires, strikes, floods, etc. The estimate of an activity's duration time should be independent of any influence that the preceding or succeeding activity may have on it. The estimates also should be based on the assumption that the resources required to carry out the activity are available. However, normal delays due to maintenance and breakdown, weather conditions, etc. should be taken care of while estimating activity duration times.

1.2 Viewing PERT in its historical perspective it was devised with a particular objective of providing a means for controlling a large-scale research and development programme. Past experience does not serve as a guide in estimating times required to accomplish research and developmental objectives. A basic element in PERT is its reliance on the experience of the person directly responsible for an activity as the best source of obtaining time estimates. The responsible person is asked to consider his activity independent of all others and based on normally available resources to provide the separate estimates of the time required by the activity. The three estimates are:

(i) Optimistic Time: (a) This is the time the activity should take if everything goes well and no mishaps occur.

Pessimistic Time: (b) This is the longest time the activity could conceivably take when everything that could normally goes wrong in series barring outright catastrophe.

(iii) Most Likely Time: (m) This is the estimate the activity should take under normal circumstances.

1.3. The three time estimates provided by the responsible person give useful information about the nature of the activity even without statistical interpretation. If suppose for an activity the a-m-b values are 5~6-7 weeks, it means that the activity can probably be completed in about six weeks time. As against this estimates of 3-10-17 weeks would mean that ten weeks remains the most likely time, but there is a very real possibility of finishing well before or well after ten weeks. The first type of activities having closely clustered time estimates are called Deterministic activities whereas the other type having wide-range time estimates are known as Variable activities.

1.4. PERT from the beginning used three time estimates, while CPM used only a single estimate. Three estimates are certainly useful in situations where high degrees of uncertainty prevails. In other situations a single estimate is un- doubted adequate. Detailed statistical treatment of these three time estimates are done later in the chapter on "Probability aspects of Scheduling in PERT". In the mean time the single estimate of mean activity duration time (t) will be derived out of the three estimates (a. m, b) by the following equation:

$$t= \frac{a+4m+b}{6}$$

This time (t) stands a fifty-fifty chance of being achieved.

2. Basic Computations

2.1. PERT and CPM use certain nomenclature for various scheduling computations. The following is a general list of such nomenclature with appropriate notations:

t = expected mean activity duration time

TE = earliest event occurrence time

TL = latest allowable event occurrence time

ES = earliest activity start time

EF = earliest activity finish time

LS = latest allowable activity start time

LF = latest allowable activity finish time

S = total activity slack (or Float)

SF = activity free slack or Float)

2.2. When three estimates for each activity time durations are used, they are converted into single time estimate by the use of the previous formula and all computations are then dealt with in the same way.

2.3 The basic scheduling computations first involve a forward and a backward pass through the network. Based on a fixed occurrence time of the initial network event, the forward pass computation gives the earliest expected start (ES) and finish (EF) times for each activity and indirectly the earliest expected occurrence time (TE) for each event. To accomplish this, the forward pass computations start with an assumed earliest occurrence time of zero for the initial project event and it assumes that each activity starts as soon as possible, i.e., the instant its predecessor event occurs. According to network logic, an event occurs when all of its predecessor activities are completed. Hence the earliest event occurrence time (TE) is equal to the largest of the earliest finish time (EF) of the activities merging to the event in question. TE is calculated by adding the time durations of all activities along the longest path leading up to that event in question. It is quite obvious that ES for any activity is equal to TE for its predecessor event and EF for any activity is equal to ES plus its duration (t).

2.4 By fixing the latest allowable occurrence time for the end event of the project the backward pass will give the latest allowable start (LS) and finish time (LF) for each activity and indirectly the latest allowable occurrence time (TL) for each event. The latest allowable start time (LS) is interpreted as the time to which the start of the activity can be delayed without directly causing any increase in the total time to complete the project. TL is computed by adding the expected times for activities on the longest path leading back from the objective event in question and by subtracting this sum from the schedule date for objective event. According to network logic an event must occur before any succeeding activities commence. Hence the latest allowable occurrence time for an event is equal to the smallest of the latest allowable start times for the activities bursting from the event in question. It is then quite obvious that LF for any activity is equal to TL for successor event and the LS for any activity is equal to LF minus its duration.

2.5 It is quite obvious that certain of the activities which lie on one parallel path (or may be more than one) having the longest cumulative time duration will govern the duration of the project. This path (or paths) is called the CRITICAL PATH. The critical path has two principal features. First, if the project has to be shortened some of the activities on that path must be shortened. The application of additional resources on other activities will not give the desired result unless that critical path is shortened first. Second, the variation in actual performance time from the expected activity duration time will be completely reflected in one-to-one fashion in the anticipated completion of the whole project.

2.6 Since critical path is the longest path from the starting to the end event, all other paths must be shorter. That means that along these paths there is time to spare, i.e. there is existing what is called TOTAL SLACK. Slack is then the difference between the earliest expected and latest allowable times. It is equal to the amount of time the activity completion time can be delayed without affecting the earliest start or occurrence time of any activity or event on the network critical path. It is a measure of f1exibility-a range of time over which the activity in question could slip without in any way influencing the accomplishment of the end objective. Slack areas are a potential source of surplus resources of men and facilities.

2.7 There is another kind of slack called the ACTIVITY FREE SLACK.

It is equal to the earliest expected time of the activity's successor event minus the earliest finish time of the activity in question. It is equal to the amount of time that the activity completion time can be delayed without affecting the earliest start or occurrence time of any other activity or event in the network. Merge point activities which lie along slack paths have this free slack. Free slack is very valuable in rescheduling activities with minimum disruption of earlier plans .

3. Illustration

3.1 The network below is for a small project with complete time estimate for each activity.

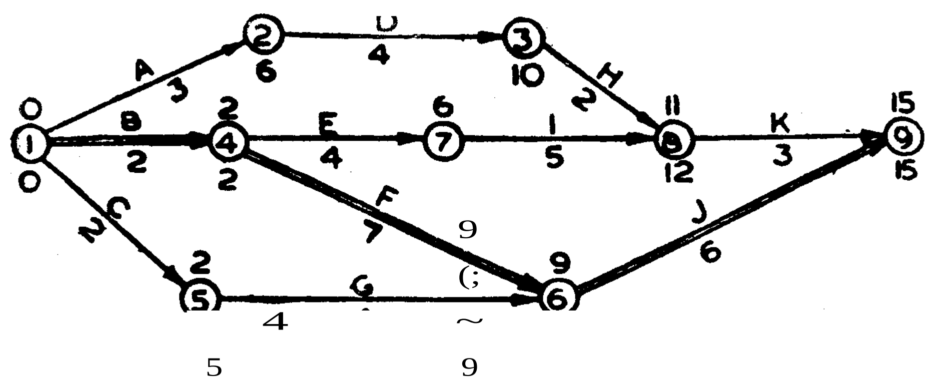


Fig.: Illustrative Network (Critical Path is shown with Heavy Line)

3.2 Following the network logic and the rules of forward and backward pass computations as detailed above. the table below gives the result with values of ES, EF, LS, LF and Slack.

Activity	t	ES	EF	LS	LF	S
A	3	0	3	3	6	3
B	2	0	2	0	2	0
C	2	0	2	3	5	3
D	4	3	7	6	10	3
E	4	2	6	3	7	1
F	7	2	9	2	9	0
G	4	2	6	5	9	3
H	2	7	9	10	12	3
I	5	6	J1	7	12	1
J	6	9	15	9	15	0
K	3	11	14	12	15	1

(i) Calculation of ES, EF :

For the initial activities A, B, C the earliest start time ES is zero. Following the rule EF=ES+t, the EF for A, B, Care 3, 2, and 2 respectively. Hence TE for the events 2, 4, 5 are also 3, 2 and 2 respectively. Then ES for D is equal to TE of event 2, i.e. 3. Therefore EF for D is ES+t=3+4=7. Proceeding this way when we come across a merge event, say event 8, we calculate the EF values for the merging activities H and I. EF for H is 9 and that of 1 is 11 Out of 9 and 11 we retain the larger value 11 which is the TE for event 8 and hence the ES for activity K. Proceeding this way all the values of ES and EF are calculated.

(ii) Calculation of LS, LF :

The largest value of EF is 15 for the activity J. This is the project duration. Assuming project duration cannot be extended the LF of J is also 15. Hence LS for J is LF-t=15-6=9. This is obviously the TL of event 6. LS for G and F is equal to TL of event 6, i.e. 9. Hence LS of F is equal to 9-7=2 and that of G is 9-4=5. In case of events like 4, where activities E and F are bursting out, the LS for both E and F are calculated separately along the two paths and the TL for event 4 will be the lower of the two values of LS. Preceding similarly all values of LS and LF are calculated.

(iii) Calculation of Slacks:

Activity total slack S is obviously the difference between EF and LF or that of ES and LS.

Activity Free Slack (SF) which is equal to the earliest expected time of the Activity's successor event minus the earliest finish time of the activity in question occurs along merge point activities which lie along slack paths. In this illustrative network SF occurs in activity G and H.

SF for *G=(TE* for event 6)-(EF of activity G) =9-6=3

SF for *H=(TE* for event 8)-(EF for activity H) =11-9=2

(iv) Identification of Critical Path:

Since critical path is the paths with the least total slack the activities B, F, J having zero slack constitutes the critical path. The other parts having slack of 1 and 3 are no critical paths and their respective slacks is a measure of their distance from becoming critical.

Sometimes in a big network identification of the critical path may be desired at the early stages of computation. Once only forward pass

Computation is complete and the values of EF and TE for all activities and events are noted on the network the following procedure will isolate the critical path at the early stages of computations without having done the backward pass.

(a) the project final event is by definition critical. Start from this event and proceed backward.

(b) while proceeding backwards when merge events are encountered, the critical path(s) follows that activity(s) for which EF==TE (Succeeding activity) .

Network analysis:
Activity-Oriented Matrix Method

1. Introduction

1.1 Projects having limited number of activities easily lend themselves for proper representation and analysis through a network by visual method. However, for bigger projects involving a few hundreds of activities initial drawing of the network and basic computations by visual method and even identification of the critical path could be quite difficult, time-consuming and sometimes confusing. Once the project has been broken down into its constituent activities with their expected time duration and the interrelationships have been found out from the knowledge of the project, the necessity of drawing a network could be obviated and all the basic scheduling computations carried out and even the critical path(s) identified by representing the data in the form of a matrix as explained below. The matrix is a more convenient tool for planning and control and is easily adaptable to changing data, facilitating the constant review of the implementation of the project.

2. Matrix Method-Illustrative Example

2.1 From the knowledge of a project let us assume that the following table (on Page 22) give the inter-relationship and the duration of the activities.

2.2 In the activity-oriented matrix method it is not necessary to draw first the network. This probably is its biggest advantage. All the information of PERT T1ME could be obtained by putting the above inter-relationships in the form of a matrix as drawn in Table].

2.3 The activities on the column heads are the succeeding activities and those on the row heads are the preceding activities. The values inserted in the body of the matrix are the activities duration values for the column head activities. From

Interrelationship table	*Activity*	*Duration*
A------------D	A	3
B------------E	B	2
B------------F	C	2
C---------G	D	4
G------------H	E	4
F------------J	F	7
E------------I	G	4
I------------K	H	2
D----------H	I	5
H----------K	J	6
	K	3

the inter-relationship table we find that the first constraint is that 0 is dependent on A. The duration of D is 4. As such 4 is inserted under the column of D against the row of A. This way the body of the matrix is filed up. After transferring all the relationships into matrix it will be observed that no entry is made under column heads A, B & C. This is because A, B, C are independent activities. This time values of A, B, C are then inserted under the column heads A, B, C against a row marked"---" which means A, B, C are preceded by nothing. Similarly as there is no insertion against row heads J & K, it means the activities J & K are succeeded by nothing.

2.4 Using the principle that an activity cannot begin until all activities precedent to it in its sequence have been completed, it is easy enough to calculate the EF values for each activity as inserted in row (i) at the bottom of the matrix. Activities A,B,C are preceded by nothing (they are initial activities); so their EF values are equal to their respective durations, as summing of course they start at 0 time. Since D is preceded by A, EF of 0 is equal to EF of A plus duration of 0 which is 3+4 =7. The value 7 is inserted below column head D in row (i). The procedure is followed column-head wise from first to last activity. One point has to be kept in mind: when a certain activity is preceded by more than one activity, the EF values for all the paths have to be calculated and the larger value retained. As in this matrix, activity J is preceded by F & G, EF for F and G are 9 and 6 respectively. The larger value 9 is retained and hence EF of J is 9+6 i.e. 15.

2.5 The largest EF value obtained in this forward pass computation (as it is called) is 15 which is the project duration. Hence the project can be programmed for completion in not less than 15 units of time. If the objective is to allow no slippage in project duration from its earliest finishing time, then the maximum value as obtained in row (i) in the matrix is also the latest finishing time for the concerned activity which must be the final activity.

SUCCEEDING ACTIVITIES

PRECEEDING ACTIVITIES		A	B	C	D	E	F	G	H	I	J	K
	-	3	2	2								
	A				4							
	B					4	7					
	C							4				
	D								2			
	E									5		
	F										6	
	G										6	
	H											3
	I											3
	J											
	K											
i	EF	3	2	2	7	6	9	6	9	11	15	14
ii	LF	6	2	5	10	7	9	9	12	12	15	15
iii	S	3	0	3	3	1	0	3	3	1	0	1

Table 1 : Activity-Oriented Matrix

2.6 Whereas the EF values are calculated proceeding from A to K column head wise, the LF values are calculated proceeding from bottom to top row head wise. This is because the only LF known at the moment of calculation is that of the last activity, i.e. activity J of which LF is 15, and it is necessary to obtain the remainder by considering preceding activities. For example, activity I precedes K. K being last activity, since it precedes nothing, must have its LF as 15. LF for 1 must be equal to LS for K which is equal to its LF minus its duration i.e. 15-3-12. Hence the LF for l is 12. The process is continued until all the valves are inserted in row (ii), below the matrix. As in the case of forward pass, in this backward pass also we have to be careful when we come across a situation where more than one activity succeeds to a single activity, e.g., activity B is succeeded by E and F. We have to calculate both the values of LF for B and take the lower value.

2.7 Finally in row (iii) the slack S is computed by subtracting EF values from LF values for each activity.

3. Conclusions

3.1 Drawing a network for a bigger project by trial and error method is quite laborious and time-consuming. This matrix method obviates all these troubles and obtains all relevant data of basic scheduling computations mechanically. Not only that, later it will be seen that this matrix is a better tool for implementation of changes in activity durations and effective project control and review.

3.2 However, the value of having a well drawn network of a project in understanding the relationships and dependency of activities should not be under, estimated.

EXERCISE

1. The interdependency relationship of a certain project is given below. The activity durations are under bracket. Develop the project matrix and calculate the values of EF, LF and S.

   (a) P (2) precedes M (8).
   (b) C (5), N (3), A (7) are in parallel and are the initial activities.
   (c) Operation E (3) follows N
   (d) C restrains the start of W (5), P, and T (5).
   (e) Job J (4) can start after A is completed.
   (f) W follows J.
   (g) Activity T occurs after E.
   (h) M (8), W, and S (2) must all be completed before X (2), the last operation can be executed.
   (i) T has to be done before S.

2. Reproduce the network of the 'Indo-Easy' project on Activity-oriented Matrix and calculate 'Slack' for each activity.
3. Find slacks for each activity of project 'RAY SEE' by activity-oriented matrix method.

Network Analysis:
Event-Oriented Matrix. Method

1. Introduction

1.1 The activity-oriented matrix method is basically a manual method and does not lend itself to computerisation for basic scheduling calculations. Therefore, activities up to 600 to 700 are handled by this method profitably. However, the event-oriented matrix is basically for electronic computer for larger scale projects. The difference between the two is that whereas the first one finds slacks for each activity the event-oriented matrix gives slacks for each event. Another difference is that in case of activity-oriented matrix, drawing of a network is not necessary while for getting the event numbers in a certain sequence drawing of the network is necessary to the extent that it facilitates event numbering.

2. Principle and Illustrative Example

2.1 The event-oriented matrix method consists of the following steps:

(i) Prepare a matrix with preceding event numbers in vertical and succeeding event numbers in horizontal column.

(ij) Divide the matrix diagonally into two parts. Insert durations on the top half of the square on the right-hand side of the diagonal proceeding from the preceding events of the activity in question up to the succeeding event.

(Hi) Note the cumulative time from the first event up to the event in question at the Jet-hand bottom of the square and carry the figure down in each square at the left-hand bottom corner, Where the figure thus carried down is different from the cumulative time already inserted the higher figure should be retained. This highest figure is then noted at the Jeft hand side of the diagonal.

(iv) Carry out this procedure to complete the left-hand side of the diagonal dividing the matrix. These figures represent the earliest event occurrence time for the respective events and should be tabulated as 'A' timings against the respective events.

(v) Now starting from the bottom. add the activity timing at the event to the number at the bottom of the column on the right-hand side of the diagonal and carry it horizontally to the left till the right-hand side of the diagonal is reached; where the figure to be carried left comes across a higher figure, the higher figure should be retained.

(vi) Carry out this procedure to complete the right-hand side of the diagonal. This gives 'B' timings which are tabulated alongside A. These B timing are essentially the um total of durations of all activities along the longest path from the event in question up to the end event. The latest allowable time for the respective events (C) would then be the project duration minus the 'B' timings.

(vii) Events for which A & C are equal fall on the critical path. For other events the value (C-A) gives the slack.

2.2 A simple network "is given below with derivation of slacks by event - oriented matrix method. .

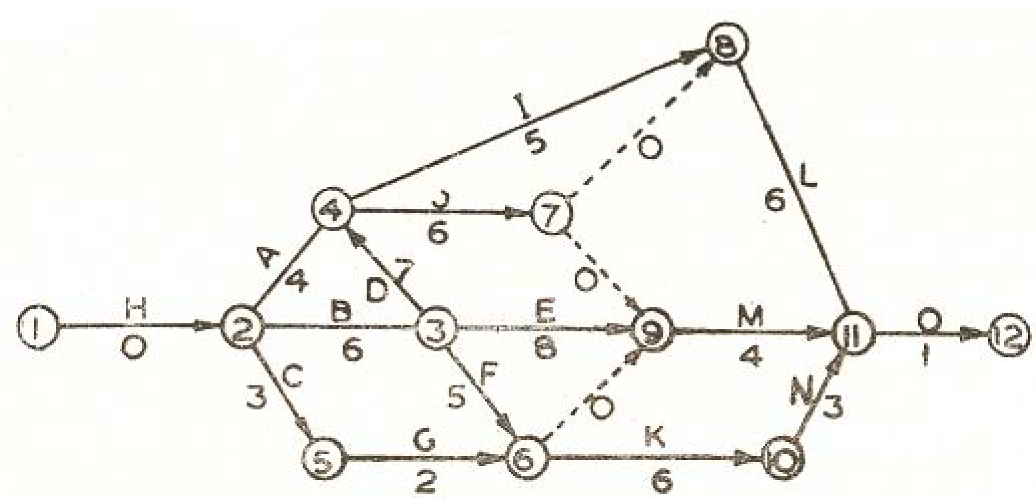


Fig. 1 : Illustrative Network

Illustrative Network

2.3 The activities are indicated by letters and their duration in weeks is shown in the network. The events are also numbered. However, in case the calculations are done by computer, it is essential that head-end event no. for the activity is always larger than the tail-end event no. The initial activity H is actually a dummy activity with 0 duration. This is added intentionally to point out that in case of computerised calculations it is required to have only one initial activity. This is achieved by the addition of a dummy in this network.

A	B	C = 26-B	Slack C-A
0	26	0	-
0	26	0	-
6	20	6	-
13	13	13	-
3	12	14	11
11	10	16	5
19	7	19	-
19	7	19	-
19	5	21	2
17	4	22	5
25	1	25	-
26	0	26	-

X = PRECEDING EVENT Y = SUCCEEDING EVENT

X \ Y	1	2	3	4	5	6	7	8	9	10	11	12
1	26 0	0 0 26										
2		26 0	6 6 26	4 4 17	3 3 15							
3			20 6	7 13 20	3 15	5 11 15	13	13	8 14 13			
4				13 13	3 13	11 13	6 19 13	5 18 12	14			
5					12 3	2 11 12	19	18	14			
6						10 11	19 10	18 10	0 14 10	6 17 10		
7							7 19	0 19 7	0 19 5	17		
8								7 19	19 7	17 7	6 25 7	
9									5 19	17 5	4 25 5	
10										4 17	3 25 4	
11											1 25	1 26 1
12												0 26

CRITICAL PATH = 1 - 2 - 3 - 4 - 7 - 8 - 11 - 12

2.4 As will be seen from the matrix the activity time durations are first inserted in the respective squares. The cumulative time from the starting event up to the event in question is then calculated and noted as mentioned on the lower left half of the square. Consider activity 2-3. Its duration is 6. The cumulative time up to event 2 is 0, as obtained from the figure inserted below the diagonal along column of succeeding event 2. Hence cumulative time up to event 3 is 0+6=6. This is then carried down the column till it reaches the lower half of the diagonal.

2.5 Consider activity 3-4. Activity duration is 7. Cumulative time up to event 3 is 6. Hence cumulative time up to event 4 is 6+7=13. This is inserted in the square and carried to the lower half of the diagonal down the column of event 4.

2.6 See activity 3-9. Duration is 8 and cumulative time up to event 3 is 6.
So the figure for the left bottom corner is 6+8= 14. This figure is carried down the column till activity 6-9. For activity 7-9, duration is O. Cumulative time up to event 7 is 19. So from 7-9 square down column 9 the largest value till activity 9, i. e., 19 will be inserted. This value of 19 is then carried up to the lower half of the diagonal.

2.7 These values which are the earliest event occurrence time is then inserted in the column 'A' left. Cumulative time up to event 12 i.e., last event is 26 which is then the project duration.

2.8 This corresponds to forward pass in the visual method. Now the cumulative time is to be determined from the last event. The cumulative time from the last event to all other events up to the first is computed from the bottom to top along preceding events and inserted at the right-hand bottom corner of the respective square. This is then carried horizontally to the left till the upper half of the diagonal.

2.9 Consider activity 7-9; cumulative time from the event 12 to event 9 is *5* inserted in the upper half of the diagonal under event 9. Activity duration 0 is added and the value 5+ 0=5 is inserted at the right-hand bottom corner.

2.10 Consider activity 3-2; cumulative time from event 12 up to event 9 is 5 as noted below column 9 in the upper half of the diagonal. Duration of activity 3-9 is 8; hence cumulative time up to event 3 along that path is 8+5==13 which is inserted on the right bottom corner of the square and carried to the left till we meet activity 3-6, cumulative time unto event 6 is 10. Duration is 5. Hence the cumulative time unto event 3 along the path is 10+5= 15. This is larger than 13 and hence retained and carried till it meets the activity 3-4 for which the corresponding value is 13+7"""20 which is then retained and carried unto the upper half of the diagonal.

2.11 The cumulative time thus obtained for each event is inserted in column B by the side of A. The project duration (26) less B times gives the latest allowable event occurrence times calculated under column C. The stack for each event is C-A.

The events for which the value (C-A) is zero are on the critical path. It is obvious then that the critical path is 1-2-3-4-7-8-11-12.

3. Conclusion

3.1 This method of scheduling computations is a little cumbersome and not generally used for manual working. The method gives an idea of computer working in giving the PERT OUTPUT in the form of the values of Ts, *TL* and slack.

3.2 One more point to be kept in mind is that the slacks as obtained here are those for the events and not for any activities. They are useful for control purposes but not for any planning of resources.

EXERCISE

1. Using the event-oriented matrix method, calculate the values of slacks for each event for the project 'INDO-EASY'.
2. Using the project memorandum of the project RAY-SEE, calculate the values of slacks for each event by drawing the network and applying visual method. Repeat the process with event-oriented matrix and compare the results.

NetworkAnalysis
SquaredNetwork

1. Introduction

1.1 In ordinary representation of a project in the network form, the activities duration times are not put in any time scale. The length and the direction of the arrow representing an activity do not mean anything. Also, the existence of slacks associated with activities is not visible from the network. To overcome these drawbacks networks could be presented in a different form where the activities flow always from left to right and their lengths are to scale representing the duration of the activities. This will automatically separate the slacks associated with each activity and they are represented by dotted lines. In squared network representation the activities are always scheduled for their earliest completion time. This facilitates rescheduling of activities when deviations from the earlier plans occur due to different reasons. A squared network combines the advantages of the usual PERT network with those of a Gantt Chart.

2. Illustrative Example

2.1 Let us take a simple network with only 6 activities with their time durations as shown below :

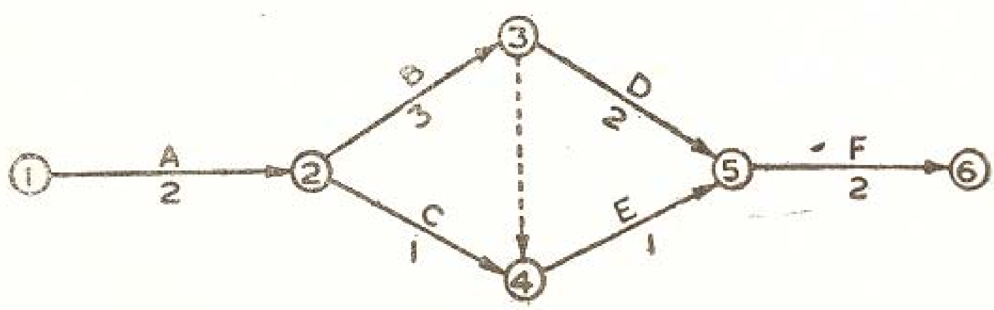


Fig. 1 : Illustrative Network

2.2 It may be noted here that drawing a squared network direct from the interrelationship chart is difficult and hence first drawing an ordinary network is necessary. For bigger project even completing forward and backward pass computations might be useful though not essential.

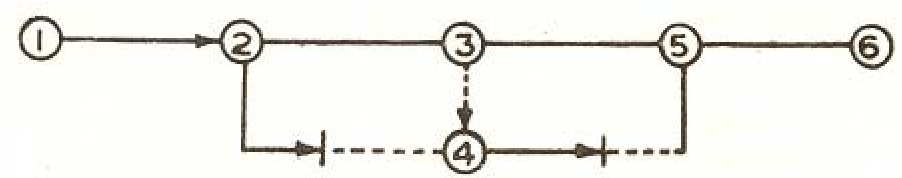


Fig 2 : Illustrated Network in Squared Network Form

2.3 For drawing time-scaled squared network, graph paper is useful. In this example, 2 squares have been taken to represent one unit of time. First, the critical path which is the longest path i.e., the path 1-2-3-5-6 is drawn to time scale. The other activities 2-4 and 4-5 are then drawn either above or below the critical path already drawn. It is obvious that the activity 4-5 cannot be scheduled immediately after 2-4 because of the dummy 3-4. The vertical dotted line 3-4 in figure 2 represents the dummy activity and since it is vertical, it represents no time duration. The squared network automatically separated the slack which for activity 2-4 is 2 units of time and for 4- 5 one unit of time. The slacks are represented by dotted lines.

2.4 In a bigger network with many parallel paths the squared network is given a good look by equally distributing the paths on both sides of the critical path to the extent possible.

EXERCISE

1. Represent the project 'INDO EASY' in the squared network form.
2. Represent the project 'RAY SEE' in the squared network form.

3. The activities for a gear box assembly project with expected time durations are given. Draw the project network from the assumed interrelationship of activities and convert it into squared network.

Activity	*Description*	*Normal time (weeks)*
A	Design	2.5
B	Drafting Check	0.8
C	drawing	0.2
D	Deliver Special materials	2.0
E	Deliver bearings, oil seals	0.5
F	Inspect purchased parts	0.1
G	Pattern for housing	2.3
H	Cast housing	0.2
I	Machine housing	0.4
J	Turn shafts	0.8
K	Heat treat shafts	0.3
L	Machine gear blanks	0.8
M	Cut gears	1.0
N	Heat treat gears	0.5
P	Assemble	2.0

Network Analysis:
Project Control It Review

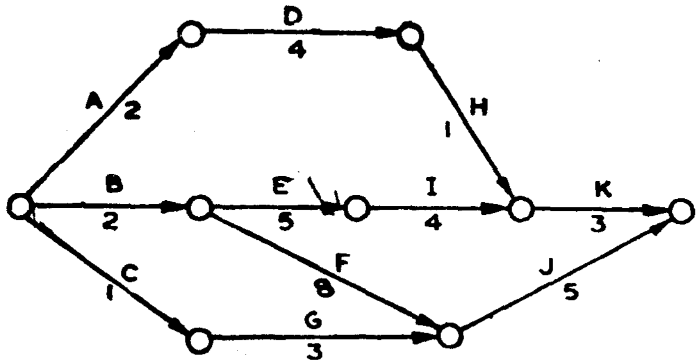
1. Introduction

1,1 It is quite understandable that in any project during implementation, the actual performance time of activities will vary from their expected duration time, factors like; late delivery repetition, due to rejection, incorrect estimates of duration and such other things could be the reasons. In such circumstances there will be a need for quick, easy and accurate method of proceeding the new programme on the basis of given changes.

1.2 One of the simplest but crude methods would be to insert the new values of project activities on the network itself and carry out the forward and backward pass computations to calculate the new values of slacks and also the shift of the critical path if any. The matrix of a project network is also a suitable means to carry out the project review due to certain changes in activity durations. Replacing the initial activity durations with the revised or changed durations and working through the matrix again will immediately give a picture of the new situation. But for a bigger project with hundreds of activities this could be laborious and time-consuming particularly when the deviations from the plan are quite frequent. In a normal industrial project one would wish to know the results of introducing one or more changes, or even a combination of changes, with the associated costs; and in those circumstances recalculation each time from basic principles will become very frustrating. A quick method of project control and review, is described.

2. Rearrangement of Data for Project Control

2.1 The first step in this method of project control is to rearrange the matrix of the project in the form of what is called a SEQUENCE TABLE. Let us take the following:



	A	B	C	O	E	F	G	H	I	J	K
-	2	2	I								
A				4							
B					5	8					
C							3				
O								1			
E									4		
F										5	
6										5	
H											3
I											3
J											
K											
EL	2	2	11	6	7	10	4	7	11	15	14
LF	7	2	7	11	8	10	10	12	12	15	15
S	6	0	6	5	I	O	6	5	I	O	1

Fig. 1 : Illustrative network and its matrix (activity-oriented)

2.2 From the matrix it is evident that the activities B,F,J have zero slack and hence constitutes the critical path. These data, however, do not indicate whether they constitute one or more than one sequence. It is convenient to be able to determine the number of sequences involved without having to refer back to the project network.

2.3 The slack values for the project activities having been calculated with initial activity durations (estimated) by the matrix method, the next step is to arrange the sequences in an order that will facilitate best and easy control of the programme. The order of importance of the sequences decreases with increasing slack values and prime consideration should be given to the 'critical' and 'near-critical' sequence.

2.4 The framework of this table is produced in Fig. 2, with row-heads representing all the activities of the project. Starting with the last activity and following the dependency relationship given in the matrix it is quite easy to locate the parallel sequences till an initial activity is reached. Care should be taken to locate the path with the minimum slack first and then proceed with the next higher slack.

A			x	
B	x	x		
C				x
D			x	
E		x		
F	x			
G				x
H			x	
I		x		
J	x			x
K		x	x	
SEQUENCE SLACK →	0	1	5	6

Fig. 2 : Sequence Table

2.5 As the first consideration is that of the critical activities, insert zero value (i.e., the slack of the critical path) at the base of the first column in the table. Select the final activity of the projects having-no succeeding activity and necessarily having zero slack and indicate its position by a "x" in the first column.

2.6 Inspection of the matrix will determine the preceding activity, which will also have zero slack. Indicate its position in the same column of the table. Continue back through the sequence until the initial activity is reached. If there is more than one critical path, there will be unaccounted slack of zero value; so the process is repeated in the second and subsequent columns of the table.

2.7 The second consideration is that of the activities with next higher slack value. It should be kept in mind that a path having a certain sequence slack, say 3, may have the activity slacks of some of the activities less than 3, but never more than 3. Examination of matrix indicates that the next path after critical pate is of sequence slack 1. One of the last activities i.e. K has slack 1. Its position is indicated. Proceeding again from K by dependency relationships the position of other activities having either 1 or zero slack is indicated constituting either 1 or more than 1 sequence, till the initial activity is reached. In this matrix, however, there is only 1 path having a sequence slack of 1. Sometimes from this point. it may be a good practice to reverse the procedure checking against the matrix rows to deduce whether there is more. than one following activity with slack value equal to or less than that of the sequence column.

2.8 In this simple manner all sequence will be deduced and indicated in an increasing order of slack. It must be stated, however, that some sequences of higher order slack might be undervalued, in that the sequence slack indicated is slightly lower than its true value. This can be rectified by checking duration totals within any sequence or can be ignored as it will not affect the results of subsequent control operations.

2.9 The process is complete when all activity slack values are considered and all sequences are indicated.

3. Simplified Approach

3.1 Having considered the more complex type of problems and its associated approach it is worthwhile illustrating a very easy method of representing relevant data for the network containing fewer activities.

3.2 Again using the network of the example, inspection quickly reveals that there are four sequences (parallel paths) involved.

3.3 These sequences may be indicated with appropriate insertion of the respective activity duration times as shown in the figure on page 37.

3.4 The total of the activity durations along one sequence gives the length of each parallel path. The largest value indicates the critical path. Subtracting the highest value from each total will give the sequence slack. The advantage in this type of sequence table is that they may be acted upon by introduction of changes, reductions or increases. As shown, the figures in column 1 of activity duration are the initial values, whereas those of column II are the changed values. The table in the right-hand side with the new values of activity duration gives immediately the picture of the effect due to changes. In this case, however, it is clear that the project duration of 15 is not effected, the critical path remains unchanged; but the sequence slack of the 3rd path, i.e. the path A-D-H-K is reduced from 5 to 2.

3.5 Hence it is realised that if one is able, with ease, to write down the sequences and associated durations, the matrix may be by-passed.

					ACTIVITY DURATION I	ACTIVITY DURATION II				
A			2		2	2			2	
B	2	2			2	3	3	3		
C				1	1	2				2
D			4		4	6			6	
E		5			5	4		4		
F	8				8	8	8			
G				3	3	3				3
H			1		1	2			2	
I		4			4	4		4		
J	5			5	5	4	4			4
K		3	3		3	3		3	3	
	15	14	10	9			15	14	13	9
	0	1	5	6			0	1	2	6

Column Totals Subtractmg Column Totals from the Highest Column Value

Fig. 3: Simplified Table

4 . Project Review

4.1 The simplified approach as discussed above is easy to handle only smaller projects with fewer activities. For bigger project~ drawing a new sequence table each time with new values of activity durations is time-consuming. The one described here and illustrated in Fig. 4 is a faster, easier and accurate method of project control and review. The net changes in the schedule and slacks due to certain deviations from expected plan is immediately known and appropriate actions could be taken care of the resultant changes.

4.2 The sequence table and the matrix are used in this method of project control and review. Each column within the body of the table contains a separate sequence and the corresponding sequence slack is given at the base row (1) and the corresponding individual activity slacks are inserted in the first right-hand side column outside the body of the table. These activity slacks are obtained from the matrix. Changes in the activity durations that have happened so far in first review of the project while the project is under implementation are inserted in the second external column. The reduction in duration is designated positive and an increase as negative. The sum of the changes in the activities along each sequence

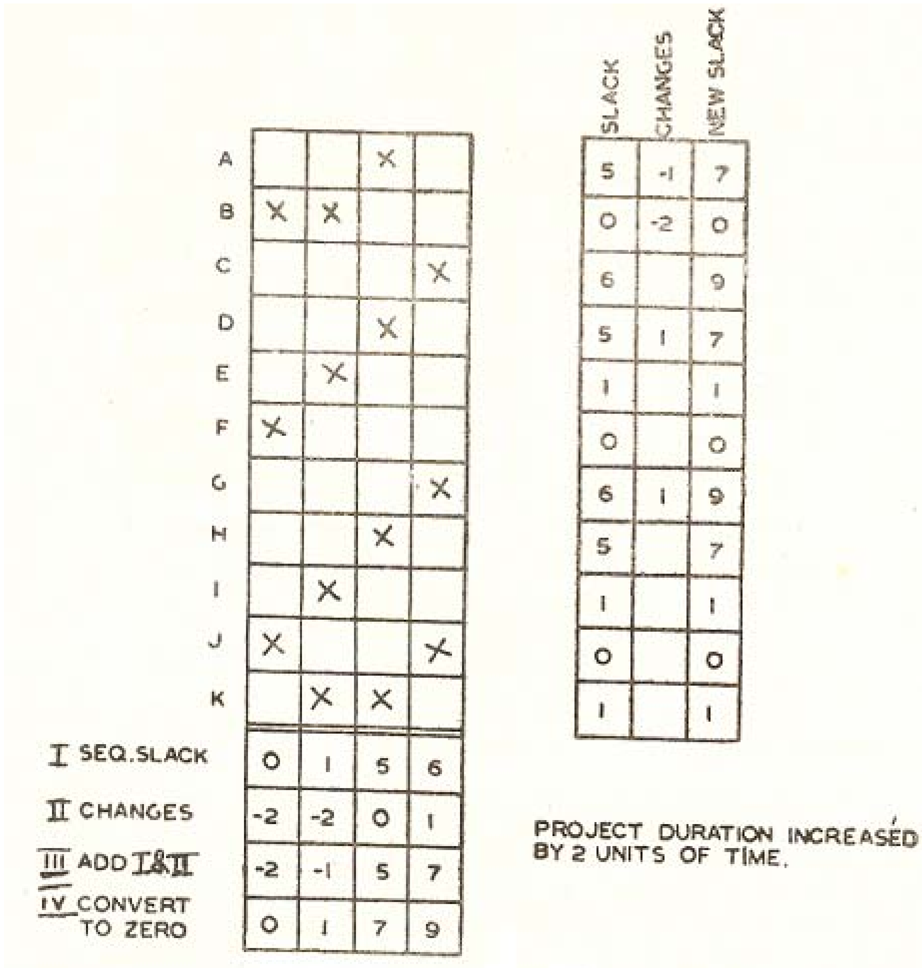


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Fig. 4: Project Review

is inserted in row (ii). For example the sum of the changes along path B.F.J. is (-2+0+0) =-2 which is inserted in row (ii). Row (i) and (ii) are then added giving new totals as in row (iii).

4.3 If all these figures in (iii) are positive and one or more is zero the new critical path is found in the column(s) indicated by zero value. If, however, there-are no zero values, or if some values are negative, it is necessary to carry out a -step called 'Conversion to zero'. This is done in the following way.

(a) All figures + ve :Subtract the minimum value in the row from each value. (Equivalent to reduction in project length).

(b) One or more values-ve : Add the positive equivalent of the minimum value in the row to all values. (Equivalent to the increase in project length).

4.4 This step will produce a new row containing only positive values with one or more zero entries (row iv). The column corresponding to the zero value contains the critical path. The other values are slacks related to their corresponding sequences. Fig.4 shows an increase in project duration by 2 units of time with certain changes in activity durations as given in second external column.

4.5 The slacks for each activity will also change due to changes in activity durations. The slacks are obtained by inspecting the new critical activities to a third external column called the 'New Slacks'. The process is continued by taking the sequence with the next higher slack value i.e. sequence BEIK. The sequence -slack, in this case **I**, is inserted against activities E, I, and K. In case of B, the value 0 is already inserted while transferring the values for the critical path. As this transference is repeated for increasing values it will be found that some lower values have previously been inserted; in these cases the higher values are ignored. The reason for this apparent duplication is the coincidence of sequences at given activities in the project; coincident sequences will have different slacks, the smallest of which will be associated with the coincident activity. In this example, B, J & K are coincident activities. Their slacks are those of the minimum values of the common sequences.

5. Conclusion

5.1 It is quite clear that this method can take care of any changes in activity durations and frequency of such deviations is no bar in immediately knowing the result of such changes on the entire project. The beauty of this technique is its capability of taking care of repetitive changes. New data can be entered and the -schedule up-dated in no time. As and when progress reports are received, the new schedule could be planned immediately and produced for distribution to the people in charge of project implementation. Correct action could be taken immediately. This is the dynamic characteristic of PERT.

Network Analysis:
Resource Constraints

1. Introduction

1.1 One of the major problems faced by management is the proper allocation of existing resources. The problem demands more attention when the particular resource is scarce or is very costly. The ultimate cost of a project will depend on how the resources are utilised; this in turn depends on project plan. The network technique makes the job simple and easier.

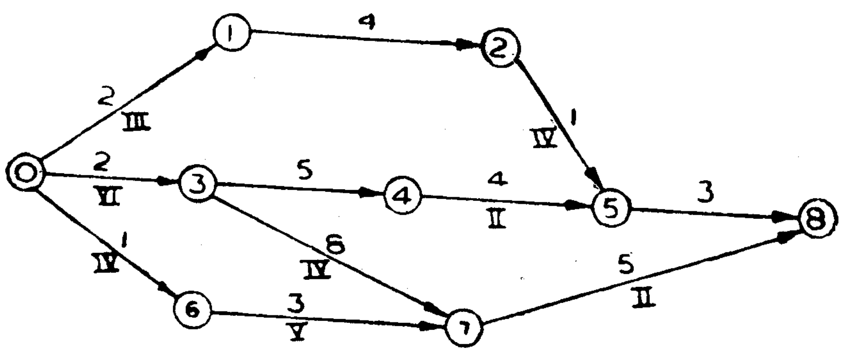
1.2 There are two basic problems involved here. One is, project duration being fixed, to level the resource demand avoiding peaks and valleys as far as possible. Other is with a fixed availability of certain key resources, to minimise the project duration. The first problem arises when one is able to procure sufficient resources to undertake the project but would like to utilise them at a relatively constant rate. The second problem occurs when one has an existing pool of resources, and would like to plan the activities so that the total project duration is minimum.

2. Systematic resource leveling procedure

2.1 The scheduling of a project consistent with a resource allocation constraint could be done in a rough manner by drawing an ordinary Gantt chart and having a cursory glance of it. However with the increasing criticality of the resources and the complexity of the problem and the frequency with which the critical resource occurs in the network, the method of resource allocation has to be more formal and sophisticated.

2.2 The network of any project being a graphical representation of its constituent activities it is quite easy to put them in the form of a bar chart with time axis after ES and EF values have been computed for each activity by forward pass computation.

Each activity could be scheduled starting at their ES values and completed at EF values. This initial schedule gives a certain pattern of resource requirements which may not be the best and has to be tested against alternative schedules. Whenever there is a slack associated with an activity, its schedule could be shifted forward and backward consistent with the restraints imposed by its preceding and succeeding activities. One such procedure based on the network of Fig. 1 is illustrated on which the required number of a certain type of special work crew is listed for each activity in Roman figures. The time unit chosen is a day.



Fig, 1 : Project Network

(Activity Crew Requirements listed in Roman letters)

2.3 If we schedule all the activities at their earliest start and finish times we obtain the bar chart as shown in the right-hand part of Fig. 2. The material presented on the left of Fig. 2 is readily available from the network on which forward and backward pass computations have been made. In this example, only one type of resource is involved. In cases where there are several different types that must be considered, then each could be listed and they should be treated sequentially from the most to the least critical. As each resource is considered it will fix the schedules of certain of the activities which are then imposed as constraints in the scheduling of the other less critical resources.

2.4 One important requirement of the left-hand part of Fig. 2 is that the activities be listed. in order of precedence. For any activity i-j, it should be seen that i is always less than j : i.e., the tail end number should always be less than the head end event number. Then one can arrange the activities in order of precedence merely by listing them in a way so that activity head numbers are in increasing order. This will ensure that all of the activity's predecessors will be found above it in the table and the successors will be below it in the table. One point should be borne in mind, that this method does not give an exclusive and unique listing. Assigning a different set of numbers to the events, still with i less than j, one could obtain a different listing of the activities.

DURATION	SCH. START	SCH. FINISH	EARLIEST START	EARLIEST FINISH	SLACK TOTAL	SLACK FREE	CREW REQUIREMENTS	ACTIVITY NO
2	-	-	0	2	5	0	3	0-1
4	-	-	2	6	5	0	-	1-2
2	0	2	0	2	0	0	6	0-3
5	-	-	2	7	1	0	-	3-4
1	-	-	6	7	5	4	4	2-5
4	-	-	7	11	1	0	2	4-5
1	-	-	0	1	6	0	4	0-6
8	2	10	2	10	0	0	4	3-7
3	-	-	1	4	6	6	5	6-7
3	-	-	11	14	1	1	-	5-8
5	10	15	10	15	0	0	2	7-8

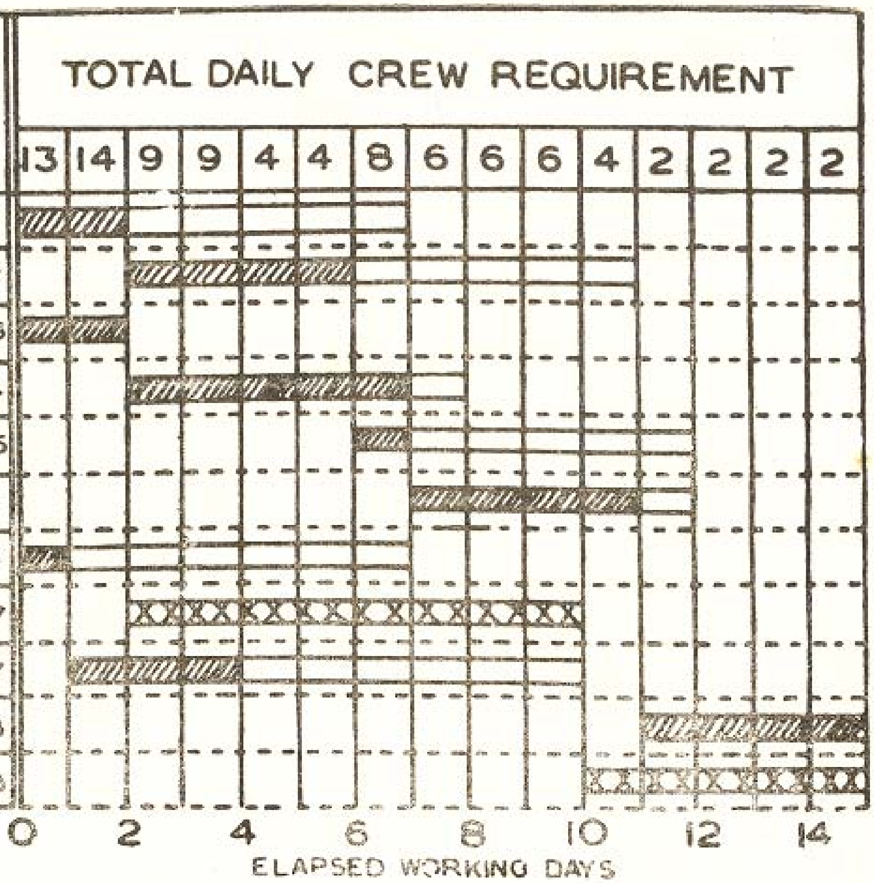


Fig. 2 : Initial Schedule of activities at their earliest times for the network shown in Fig. 1

Different listing might result in different schedules for the particular resource constraint and if the particular resource is a very critical one it may be worthwhile to try with different listing and to choose the best result.

2.5 From the bar chart drown in Fig. 2 the daily crew requirement could be ascertained readily. For example, the requirements during the first day are 4 (for activity 0-6)+6 (for activity 0-3) + 3 (for activity 0-1)= 13, which is entered at the top of the bar chart. This particular earliest schedule then requires a maximum of 14 crews, which occurs on the 2nd day. It also has a considerable variation in resource requirement from a maximum of 14 to as low as 2. Obviously one can improve on this schedule. Activities 0-3, 3-7 and 7-8 cannot be altered because they lie on the critical path. But by sliding the slack activities back and forth within the available total slack, one can obtain an improved schedule.

2.6 To systematise this scheduling procedure a method was developed which compares alternate schedules using the sum of the squares of the resource requirements during each time unit as a measure of effectiveness. When the variation in the resource requirement is leveled to the extent possible this sum of squares would be the minimum. The effect of alternate schedule on a two-day assignment which requires eight man days of effort in any combination is shown in figure 4.

2.7 Applying this concept to the schedules in Fig 2 and 3, the total sum of squares would be :

Fig. 2: $13^2+ 14^2+9^2+9^2+4^2+4+8^2+6^2+6^2+6^2+4^2+2^2+2^2+2^2+2^2=763$

Fig. 3: $6^2+6^2+7^2+7^2+8^2+9^2+9^2+9^2+6^2+6^2+4^2+8^2+2^2+2^2+2^2=641$

2.8 The following are the successive steps of activity scheduling procedure to level resource requirements:

Step 1 : List the activities by arranging the arrow head numbers in ascending order and for merge events, list the activities with tail end numbers in ascending order, Prepare a bar chart as in Fig. 2, with earliest schedule of all activities and indicate the associated slack. If some activities have fixed schedules due to previous resource constraint or due to being critical activities, they should be drawn with no slack associated to them.

Step 2: Starting with the bottom activity i.e., the last activity, schedule it to give the lowest total sum of squares of resource requirements for each time unit. If two alternatives give the same total sum of squares, , then schedule the activity as late as possible, thereby releasing slack for all preceding activities.

Step 3: Taking advantage of the slack releases in step 2, repeat step 2 for the next to the last activity, holding the last activity fixed. In general, by observing the scheduled start time of the activities found below the one in question, the slack availability check is made and the earliest of these observed scheduled start times is then the latest allowable finish time of the activity in question.

Step 4: Step 3 is continued till the first activity is reached. This completes the first rescheduling cycle.

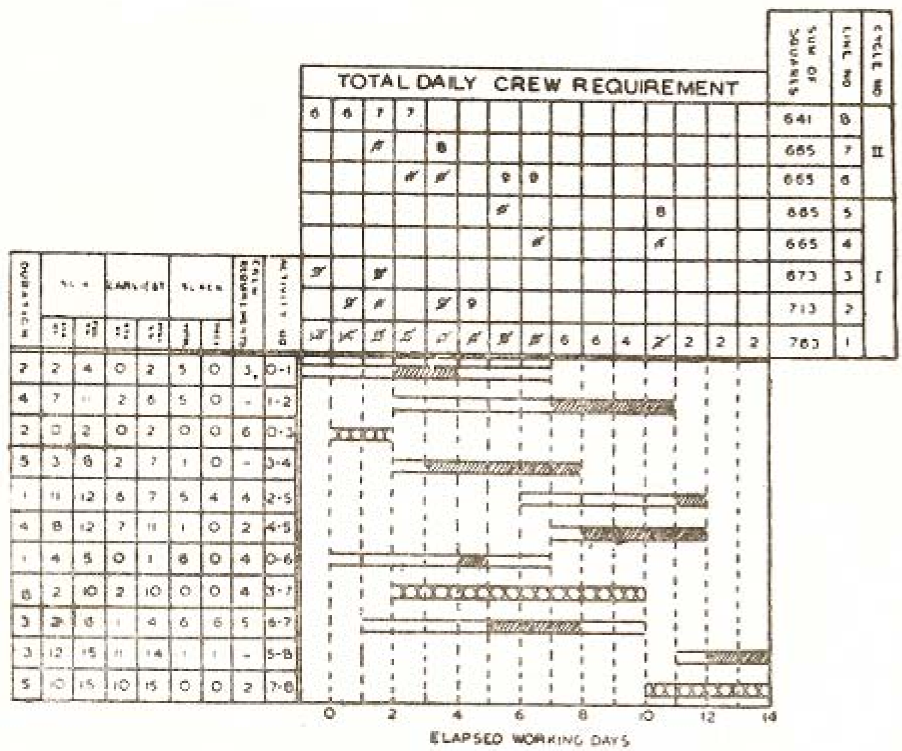


Fig. 3 : Optimal Schedule of activities for network of Fig. 1

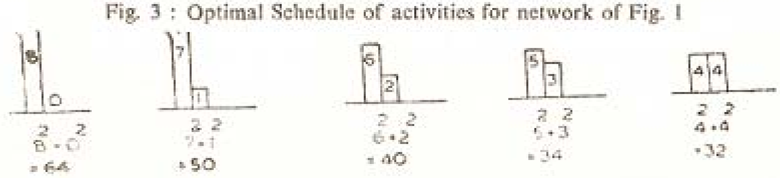


Fig. 4: Effect of various Schedules on Total Sum of Squares of daily requirements

Step 5: The cycle is repeated from step 2 to 4 until no further reduction In the sum of squares of resource requirements is possible.

Step 6: For a particular critical resource, repeat steps 1 to 5 with different numbering of activities listing the activity still in order of precedence.

Step 7 After steps 5 and 6 are complete, choose the best schedule.

Step 8: Final adjustment is then made to the schedule taking into consideration factors not already considered.

2.8.1 In carrying out step 2, if one moves an activity one day at a time, only the activity end effects need be considered. For example, if activity 4-5 in Fig. 2 is moved one day to the right the total sum of squares is affected only at the two 'ends' of this activity, i.e., on day 8, which is being vacated and on day 12 which is being occupied.

	Current schedule	*New schedule*
Day 8 requirement	6	4
Day] 2 requirement	2	4
Sum of Squares	$6^2+2^2=40$	$4^2+4^2=32$

2.9 The Total requirement for each schedule is the same, i.e., 6+2=8 and 4 +-4=8: however, the total sum of squares is less for the new schedule, hence in this case we would move activity 4-5 one day to the right, and reduce the total sum of squares by 40-32=8.

2.10 Fig. 3 shows the application of this procedure. In this example the procedure terminated after two rescheduling cycles. Table 1 gives the successive steps carried out in Fig. 3 for minimising the sum of squares gradually. Line I gives the requirements of the crew as per initial schedule. Working up the list of activities one observes that activity 7-8 is critical and hence cannot be moved. Next, activity 5-8 is moved to the right one day but, does not require any of the crews. Next activity6-7 is moved one day to the right with a reduction 50 in the sum of squares and then another day to the right with no change in the sum of squares. However, further move to the right would increase the sum of squares. Necessary changes are then made in line 2 after shifting activity 6-7 by 2 days to the right.

2.11 The final crew requirements for each day as per Fig. 3 is as follows; 6, 6,7, 7, 8,9,9,9,6,6,4. 8,2, 2, and 2. Before this schedule is finalised, some adjustment may be made as suggested in step 8, to take into account some factors not already considered. In case of activities -2 and 3-4, nothing has been gained In scheduling them as ate as possible and hence they could be moved back to their

TABLE 1

Line in Table	Cycle	Activity Schedule Change	Sum of Squares
1	I	all activities at their earliest times	763
—	I	activity 5-8 from 11-14 to 12-15	—
2	I	activity 6-7 from 1-4 to 3-6	713
3	I	activity 0-6 from 0-1 to 2-3	673
4	I	activity 4-5 from 7-11 to 8-12	665
5	I	activity 2-5 from 6-7 to 11-12	665
—	I	activity 3-4 from 2-7 to 3-8	—
—	I	activity 1-2 from 2-6 to 7-11	—
6	II	activity 6-7 from 3-6 to 5-8	665
7	II	activity 0-6 from 2-3 to 4-5	665
8	II	activity 0-1 from 0-2 to 2-4	641

mes to conserve their slack. Also it would be better to move activity 2-5 back from day 12 to II so that the final crew requirement will taper off in a more desirable manner, i.e., 6, 6, 8,4,2,2,2 instead of. .. 6, 6, 4, 8, 2, 2,2.

3. Extension of the problems of resource constraints

3.1 There are projects which contain groups of activities (cycles) that are repeated a number of times. Projects like road construction, multi-storeyed buildings, bridges etc., are the examples. Since the objective in a single work cycle is to complete it/repeat it as soon as' possible, in case of repeated work cycles, it is obviously desirable to repeat it as soon as possible. Only logical condition 'would be that anyone activity must be completed in cycle before the same activity can be started on the next cycle. Drawing a bar chart is in the case of Fig. 2 and repeating each activity as soon as possible it becomes clear that the shortest cycle time will always be limited by the longest activity in the cycle. If Fig. 2 represents just one cycle *if* work that is to be repeated a number of times the shortest cycle time will always be limited by the longest activity in the cycle. If Fig. 1 represents just one cycle of work that is to be repeated a number of times the shortest cycle time becomes an eight-day cycle. The systematic resource leveling procedure can then be again applied on this repetitive schedule to arrive at the optimal result.

3.2 Another resource allocation problem is to determine whether a certain specified resource is sufficient to man the project activities or not. If not, certain activities must be delayed (duration extended) in such a way that the specified resource availability is sufficient and also the total project duration is increased by a minimum amount. In such problems activities having slacks and requiring the started resource are first extended to minimise the resource requirement. If this is not sufficient then the activities on the critical path(s) are studied to find the possibility of maximum reduction of the resource requirement per unit increase in project duration.

3.3 This process of stretching out the duration of critical path activities and then slack path activities must be continued until the total resource requirement is equal to the available limit.

3.4 Another extension of the resource allocation problem is the comprehensive multi-project scheduling in which a firm undertakes multiple projects, simultaneously drawing on the same available limited resources. The objective in this case is to schedule each activity so that (1) project completion dates are met or their over-runs are minimised, (2) resource requirements do not exceed stated availabilities and (3) idle resources are minimised. RAMPS (Resource Allocation and Multi-Project Scheduling) which is a computerised system is basically designed for such comprehensive multi-project scheduling.

EXERCISE

1. Suppose the time estimates for the activities making up the project network in Fig. 1 in this chapter were based on the assumption of carpenter crew assignments given below :

Activity	*No. of Crews*	*Activity*	*No. of Crews*
0-1	2	4-5	4
0-6	4	6-7	4
0-3	6	7-8	5
3-7	4	all others	0

Prepare the project schedule so that the largest number of crews required at anyone time is a minimum.

2. For the example 1 above, find the total project duration when the maximum crew availability is 8 only. If the availability of crew is only 6, bow far the project would be stretched?

Optimisation of Cost:
Network Cost Control

1. Introduction

1.1 The extension of the PERT concept to include cost considerations was a natural One. It is not enough only to complete a project on time. This must be achieved with optimum cost. The amount of priority that should be given to the achievement of a certain project by a certain date depends on its cost consideration. It could be worthwhile to give more time for the whole project if the resultant savings are considerable. Conversely One should know the extra cost involved in expediting a project by a certain amount of time.

2. Time-cost relationship

2.1 Activity direct costs include the costs of material, equipment, and direct labour required to perform the activity in question. If the activity is being performed in its entirety by a sub-contractor, then the activity direct cost is equal to the price of the sub-contract. Project in direct costs may include, in addition to supervision and other customary overhead costs, the interest charges on the cumulative project investment, penalty costs for completing the project after a specified data, etc.

2.2 The 'normal activity cost' is equal to the absolute minimum of direct costs required to perform the activity, and the corresponding activity duration is called the normal time. It is this normal time that is used in the basic CPM/PERT planning and scheduling and the normal cost is the one usually supplied if the activity is being sub-contracted. The normal time is actually the shortest time required to perform the activity under the minimum direct cost constraint i.e., this rules out the use of overtime labour or special time-saving but more costly material or equipment.

2.3 OrdiJl.3rily a project would not be scheduled for a longer duration *that* the normal ~l1ration time. However, for various reasons one may be interested in replaning and scheduling the project for a shorter duration time. Selection of a certain schedule for a project generally involves a comparison of a number of alternatives, each with a different cost picture. In most activities there is a direct relationship between cost and the time of completion of the activity. A certain activity could be expedited (duration shortened) by incurring extra cost in the form of assigning more personnel or scheduling overtime for that activity. When an activity is completed in normal scheduled time, the associated minimum cost could be called 'normal cost'. When an activity is completed in minimum possible time the associated time could be called 'Crash time' and the cost 'Crash cost'. An activity could be scheduled for any time between these normal and crash time with a cost value lying between normal cost and crash cost. Actual cost-time relationship curve could be of any shape, but mostly they are linear. In certain cases there may be only few feasible time-cost trade-offs between normal and crash points. However for optimisation problems, we would assume the relationship to be linear with a constant cost slope.

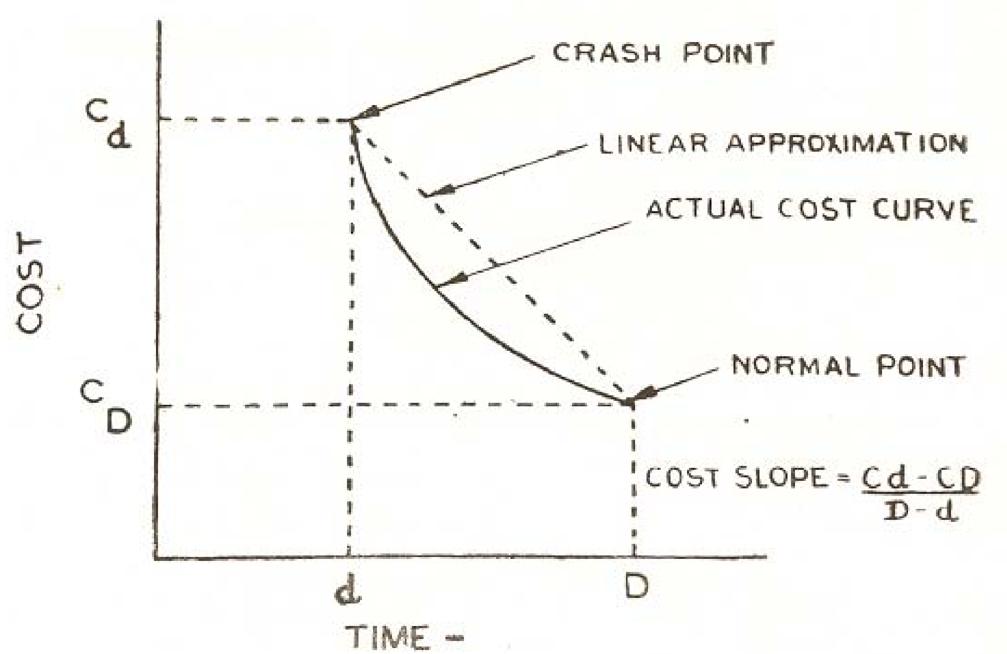


Fig. 1 : Activity time-cost relationship

3. Project reduction at minimum cost

3.1 The normal cost for a certain project could be obtained simply by summing the normal cost for all the activities of the network. In order to shorten the project duration with minimum extra cost, the critical path is first examined and the activity with the minimum cost of expediting is first shortened. As the total critical path duration is shortened gradually, other paths in turn might become critical and they too have to be examined. Sufficient points could be plotted in this way to show the variation of direct project costs with time (Project duration). Addition of the indirect-cost patterns to this curve gives all the information a manager requires to select the project schedule balancing the cost and time objectives.

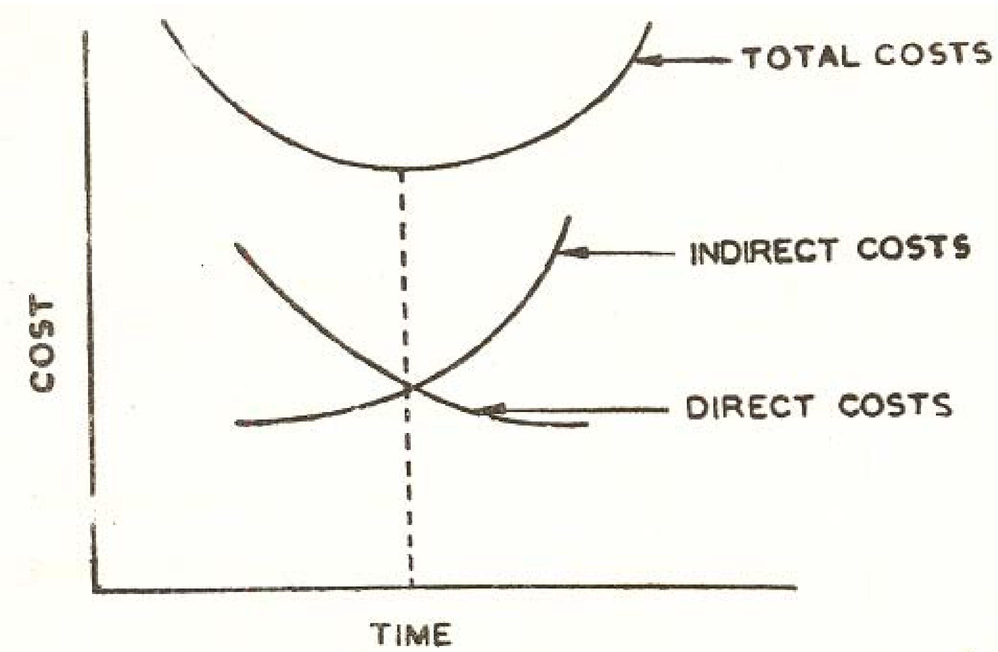


Fig. 2: Total Project Costs

3.2 To illustrate the procedure let us take the simple example which has the network and the relevant cost as follows:

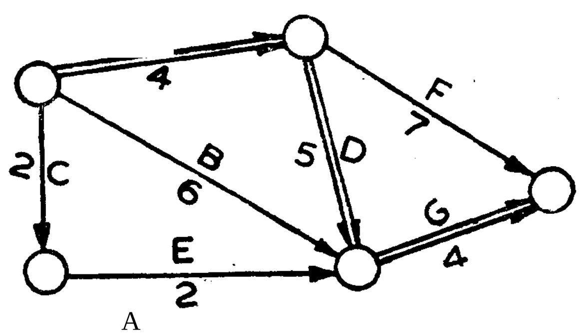


Fig. 3: Illustrative Network with Cost Data

Activity	Normal		Crash		Cost Slope Crashing Cost Days
	Days	Cost	Days	Cost	
A	4	60	3	90	30
B	6	150	4	250	50
C	2	30	1	60	30
D	5	150	3	250	50
E	2	100	2	100	—
F	7	115	5	175	30
G	4	100	2	240	70
	Total	705			

3.3 To find the cost slope (figures in the last column) one has to find the difference between Normal Cost and Crash Cost and divide it by the difference between normal time and crash time .

Therefore,

$$\text{Crashing Cost/Day} = \frac{\text{Crash Cost-Normal Cost}}{\text{Normal Time-Crash Time}}$$

3.4 This, however, is on the assumption that cost-time relationship is linear. The data shows that activity E is one which must take a minimum time of 2 ~)!S and cannot be hurried for technical/safety reasons no matter what one is willing to spend. The crashing cost may be in the form of providing staff fora.4ditional shift, advancing delivery dates of material, purchasing items 10Ca.1Iy'.0f expensive transport charges due to air freighting etc. The days shown in the crash column are the minimum it takes to do the jobs.

3.5 The critical path is obviously A D G and the normal project duration is therefore 4+5+4=13 days and it corresponds to the minimum direct projects cost ofRs.705.

Step 1: To shorten the project duration, we have to shorten the critic8;I' path. Out of the activities A, D & G the cheapest is to shorten activities A; the time available for crashing is I day for an additional cost of Rs. 30.

Hence project duration =13-1=12 days.
Total direct cost =705+30=Rs. 735.

Step 2: Activity A having crashed to the maximum the next step would be to crash either D or G. Out of the two it is cheaper to crash D with an extra cost of only Rs. 50/- per day. Time available for crashing is 2 days. If we crash by 2 days, then

Project Duration

Total direct cost =12-2=10 days. =735 + 100=Rs. 835.

Step 3 : After step 2 is completed two separate paths AF and ADG become critical, both having the duration of 10 days. In crashing the project further the only alternative is to crash both F and G simultaneously, and the additional cost for crashing one day would be 30+70=Rs. 100/-.

Then Project Duration =10-1=9 days.

Total Direct Cost =835+ 100=Rs. 935.

Step 4 : From the data we find the time available for crashing for both F and G is 2 days out of which 1 day is crashed in step 3. If the remaining 1 day is crashed the extra cost would be another Rs. 100/-.

then Project Duration=9-l=8 days.

Total Direct Cost =935+ 100=Rs. 1035.

3.6 Examination of the data shows activities on both the critical paths have been crashed to their minimum duration and crashing the activities B, C or E will have no effect on the project duration

3.7 In the above example, the project is compressed to a limit. Without regard for east. That is unlikely hi actual field beastie the gain in. getting the plant into production sooner might well be offset by the additional cost of crashing the job. To show how this indirect consideration can affect the schedule decision, certain arbitrary figures have been assumed for indirect expenses (overhead). The best schedule is one which has lowest total cost.

Project Duration	Total Direct Cost	Total Indirect Cost	Total Cost
13	705	400	1105
12	735	250	985
10	835	100	935
9	935	75	1010
8	1035	50	1085

Hence it is obvious that a 10-day schedule is the best and most economical.

4. Network cost control

4.1 In theory the concepts of cost control based on the project., network, is simple but the design and the implementation of a practical cost control system is not readily accomplished. The basic problems facing the designer of cost control system based on network approach are: (a) those related to orsanisatioual conflicts and (b) those related to the necessary efficiency of the system. The basic organisational problem is the conflict between the project approach of network cost control and the functional approach of cost accounting procedures found in most .of industries. The input to a network system requires the development of an activity accounting procedure by which actual expenditure data are coded to provide association with activities in the project network. The output for the system likewise must be project-oriented to provide project summary reports, organised by time period, areas of responsibility, and technical sub-division~ of the project.

4.2 The efficiency of the system is also a problem because the level of detail is simultaneously a promise as well as an inherent hazard. A network cost control system can easily require routine input data in quantities and frequency that project personnel find extremely burdensome. Unless the requirements are reduced and procedures simplified, the system will come to an early end.

4.3 The PERT/Cost procedure requires as input cost data in additive in to the time data required by baste PERT. This cost data is generally collected for amatol groups of related activities rather than for single activities, so as not to impose an undue accounting burden. Cost estimates are obtained only after a satisfactory schedule has been developed, since any schedule change will normally affect cost. As the project progresses, actual accrued costs are gathered for each cost collection point and revised estimates are submitted as required. A number of useful and informative reports can be generated from this data. The basic output is a status report, which combines time and cost data for each cost collection point. This enables the manager to identify activity groups which are contributing to actual or potential schedule slippages or cost over-runs and also to compare the time and the cost status of any given activity group. In addition to the out put .obtained from time-oriented network, this report shows the original cost estimate, the actual costs incurred, a revised estimate, if any, and the anticipated under-run .of over-run. Provision is made for summarisation of the time and cost 4ata at various levels, so that each level of management is presented with only that amount .of detail with which it is directly concerned.

4.4 Upon receipt of all input data on the pr6itess and expenditure as of the reporting data, and any revised estimates of durations or costs of activities not completed, the following computations may be made:

(i) Summation of all actual costs

(ii) Summation of budgeted costs at this point in time

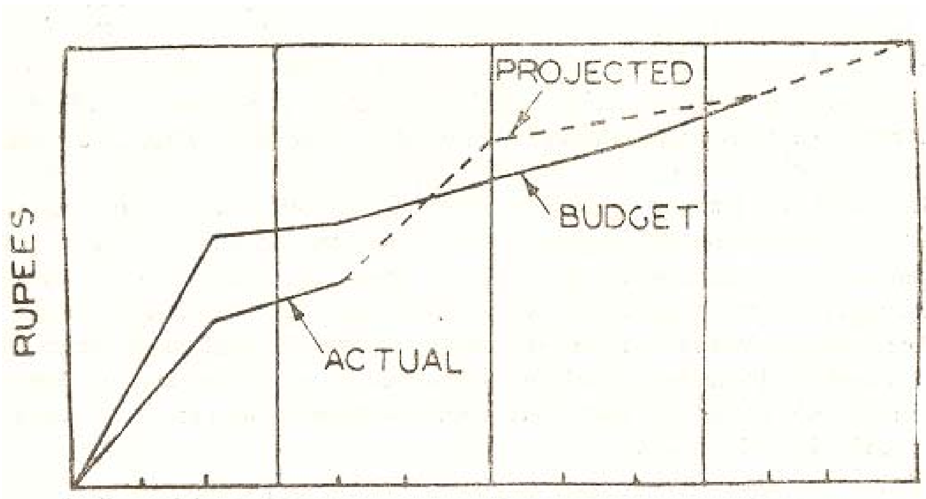
(iii) Summation of budgeted cost for all activities completed and proportional cost of activities partially completed. This figure is called 'planned cost of work completed' or 'value of work'.

(iv) Computation of difference in actual cost and planned cost of work completed computation of both money value and percentage of planned cost of work completed.

(v) Computation of a projection of ultimate cost of the total project.

4.5 The sum of actual of date and latest revised estimates of future cost may then be compared with the contracted total cost for the project, to indicate the amount of the expected over-run or under-run.

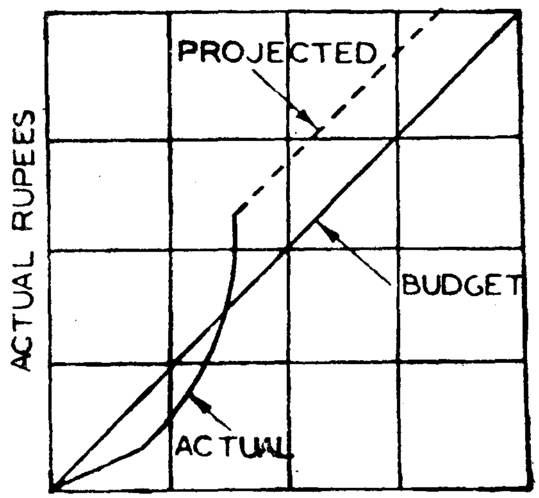
4.6 A good appreciation for the advantages of PERT/Cost over conventional budgeting and cost control techniques can be gained from Fig. 4. Using conventional techniques, departmental budgets are usually based on calendar periods. From this point of view, the costs of the project depicted would appear to be well under control at the end of the fourth month, since the It lonely actually spent fall well below the money budgeted for the period to date. However, since funds are budgeted by units of work to be performed under a PERT/Cost system, PERT/Cost clearly shows that if past experience is projected into the future a time over-run of two months can be expected along with a serious cost over-run.



MONTHS

Fig. 4; Rate of Expenditure

4.7 Fig. 5 compares cumulative budgeted and projected costs in terms of the amount of wo~k performed. This graph is essentially useful in evaluating the amount' of over-run already encountered, the trend established and the probable total over-run.



BUOGETED MONEY

Fig. 5 : Cost of Work

EXERCISE

1. The time and cost data for the activities of a Gear Box Assembly Project are given. Identify the critical path on normal time estimates. Crash the project to its minimum duration and find the associated increase in cost.

	Normal			*Crash*		
Activity	*Timer weeks*	*Slack l weeks*	*Cost Rs.*	*Time l weeks*	*Cost l Rs.*	*Cost Slope RI.I week*
A. Design	2.5	0	450	1.5	.700	250
B. Drafting	0.8	1.0	140	0.5	200	200
C. Check Drawing	0.2	1.0	35	0.1	45	100
D. Deliver Special materials	2.0	.0	10	1.0	30	20
E. Deliver Bearings, oil seals	1.5	3.3	10	0.5	20	10
F. Inspect purchased parts	0.1	3.3	20	0.05	25	100
G. Pattern for housing	2.3	.0	350	1.3	550	200
H. Cast housing	0.2	.0	50	0.1	75	250
I. Matching housing	0.4	.0	100	0.3	150	500
J. Turn Shafts	0.8	1.8	175	0.3	375	400
K. Heat Treat Shafts	0.3	1.8	75	0.3	75	
L. Machine gear blanks	0.8	0.6	175	0.4	325	375
M. Cut gears	1.0	0.6	250	0.5	450	400
N. Heat treat gears	0.5	0.6	125	0.5	125	
P. Assembly	2.0	.0	300	1.0	600	300

2. In project RA YSEE use cost data to determine optimum time and cost in a crash problem.

Product Reduction at Minimum Cost: Matrix Method

1. Introduction

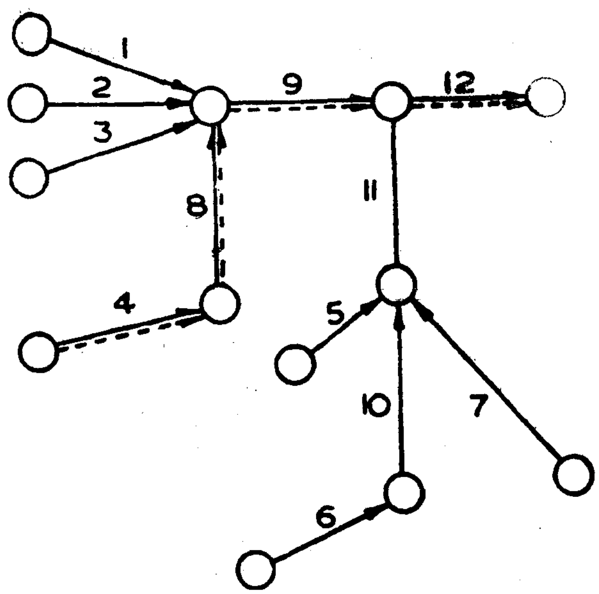
1.1 Due to enforced or accidental changes in activity durations there is a corresponding change in the project duration which, we have already seen, could be determined with ease and accuracy. Given the cost data for each activity i.e., the normal cost. crash cost and the slope it is also possible to optimism the cost due specific change in project duration by all analysis of the network. However, this Chapter discusses the approach and methodology in reducing the project duration at minimum cost possible by utilising the project matrix. One thing should be borne in mind that it is immaterial whether the cost slope is linear or not, only that it should be possible to give a value to the increase in cost for specific reduction of any activity duration. The illustrative example, however, is worked out assuming. that the cost slope is linear.

2. Illustrative Example

2.1 In order to demonstrate the likely scope of use of the matrix for Cost reduction, let us consider the project network given in Fig I.

2.2 From this network the procedure is as before, with the construction of the matrix as shown in Fig. 2, and sequence table. For simplicity the cost increases for reductions are assumed to be linear and may be listed as 'rates' associated with the activities within the table as shown.

2.3 If the project is to be decreased by one week, the only sequence worthy of consideration is the Critical Path and the obvious choice for reduction is the activity with the lowest extra cost rate, namely activity 4. The same argument



Activity No.	Duration in weeks
1	3
2	1
3	6
4	5
5	4
6	1
7	1
8	4
9	2
10	5
11	3
12	2

Fig. 1: Illustrative Project Network

applies to a two-week reduction, as the next parallel sequence has a two-week slack: hence the activity reduced is again activity 4 for two weeks.

2.4 In order to effect a three-week reduction, a choice needs to be made; either reduce common activity 12 by three-weeks or reduce an activity from the first column by three weeks and an activity in second column by one week (available slack 0, and 2 respectively). Inspection of costs quickly indicate the advisability of choosing the second possibility, with reduction in minimum cost activities.

2.5 However, when considering a reduction of, four weeks a new alternative is presented and the choice is between reducing:

(a)	Activity 12 by four weeks at cost	Rs.	200
(b)	Activity 9 by four weeks	Rs.	80
	Activity 6 by two weeks	Rs.	50
		Rs.	130
(c)	Activity 4 by four weeks	Rs.	40
	Activity 6 by two weeks	Rs.	50
	Activity 9 by one weeks	Rs.	20
		Rs.	ll0

2.6 But closer inspection of (c) above reveals that as activity 9 is common to the first and third column, the additional reduction necessary in the first column is only three weeks; hence (c) becomes:

Activity 4 by three weeks	Rs.30
Activity 6 by two weeks	Rs.50
Activity 9 by one week	Rs.20
	Rs.100

NOTE: Activity 6 has a normal duration of one week, but it has been reduced by 2 weeks. This anomaly has happened since these considerations are not taken into account. However, project reductions with limits imposed are explained below:

2.7 The choice of reductions shown above raises a new probability, namely that were will be a limitation in the degree of reduction possible on every individual activity. These limitations could be due to anyone of many factors, physical constraints of the activity, and restrictions by external suppliers. Unavoidable durations in transit, etc.

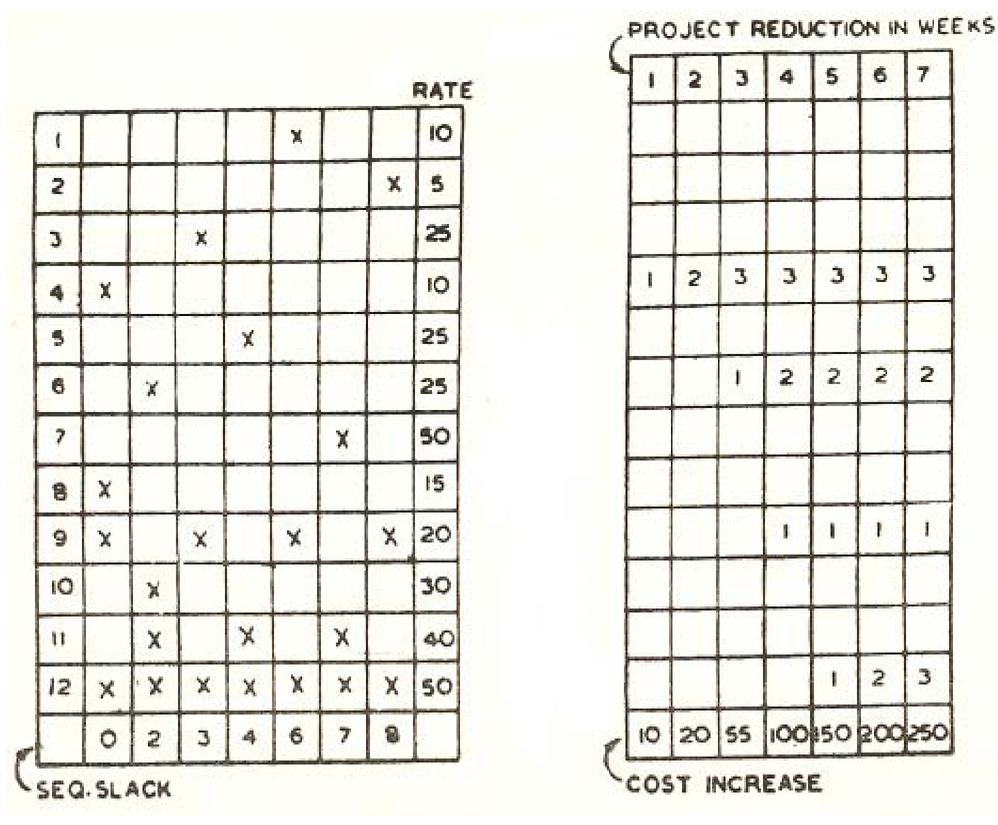


Fig. 2: Reductions at Minimum Costs (Without Limits)

2.8 Consider the new sequence table (Figure 3) showing the normal and crash durations for each activity. The reductions in project duration and their associated costs are the same as in the example quoted above up to the extent of two weeks reduction. A project reduction of the weeks was previously possible through three weeks decrease in activity 4, and a one-week decrease in activity 6; now, however, activity 4 is limited to a two-week reduction, hence the third week must be taken from the activity with the next cheapest rate, namely activity 8. Moreover, activity 6 cannot be reduced, hence the activity to be reduced becomes activity 10. With such limitations in mind, each sequence is considered as necessary until the pattern of reduction associated with the minimum cost increase is achieved.

2.9 A method of confirming the results obtained and also of arriving quickly at the solution for a given overall reduction is now illustrated more fully.

2.10 If there is to be a reduction of the duration of the project, there is usually a choice between a numbers of possible individual activity reductions. This choice is limited by the necessities of the sequences in the project, together with the degree of reduction permissible for individual activities. However, not with

standing this, there will usually be a selection of patterns, each with the associated increase in cost. The requirement is to choose the pattern that incurs the smallest increase in cost while being within the boundaries of constraints.

2.11 The sequence table of Figure 4 has a slight variation on previous examples, in that, for any sequence, the activity indication (x) has been replaced; by the cost of reducing that activity for one tile unit (where it is not permitted, to reduce, then 'x' is retained). These values are shown as small figures in the body of the table. It is obviously quite easy to produce a reduction pattern conforming with the conditions of the current situation and the table illustrates this for an overall reduction of six weeks, where the large figures in the body of the table represent individual reductions. This, however, is not necessarily of minimum cost.

N.B. - for column 3 with available slack three weeks, the aggregate reduction must be three weeks; this is made up of the two-week reduction on activity 3 and the reduction of one week in activity 9 as indicated by the first column.

2.12 It is now necessary to change the given 'Feasible solution' for one that i$ more economic. Consider the final relevant column (Column 4).

2.13 Insertion of positive unit in common activity 11 suggests an increase in cost of 40 units but there is reduction possible from activity 5 of 25 units and from activity 10 in associated column of 30 units; hence an overall reduction of 15 units. This then is the 'First Change', showing as it does, a real improvement.

2.14 Repeated inspection indicates that a positive insertion for activity 12 will add 50 units but by selective reductions, a value of 95 may be subtracted, producing an overall saving of 45 units. Further examination reveals that there is no possibility of added savings and hence the final solution is obtained.

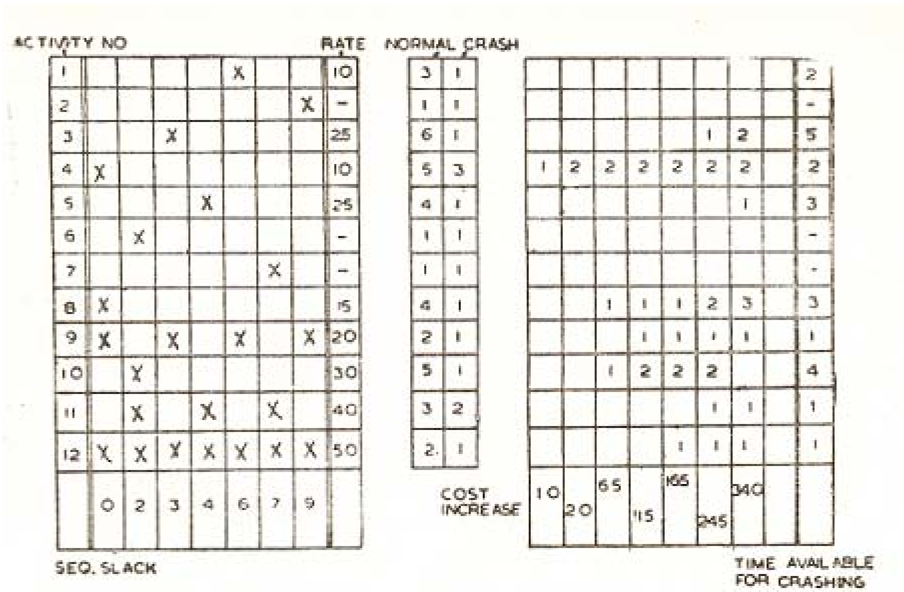


Fig. 3 : Reductions at Minimum Costs (limits imposed)

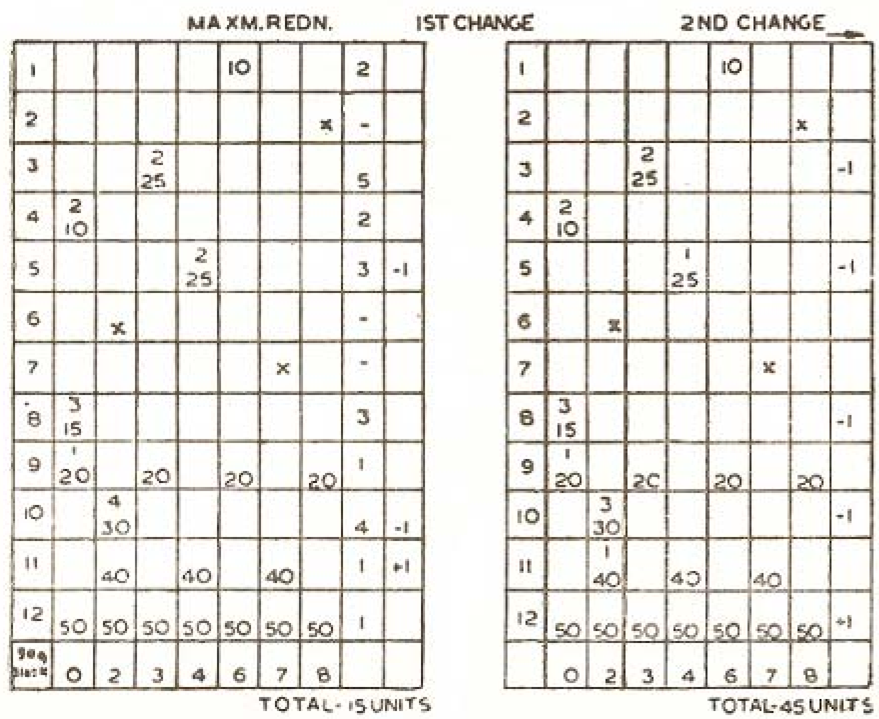


Fig.4:CostReductionMatrixMethod

Probability Aspects of scheduling in PERT

1. Introduction

1.1 All scheduling systems so far are based on a fixed time estimate for a certain activity. But these estimates are subject to considerable chance sources of variation, such as the weather, equipment failure, absenteeism and uncertainties in the methods and procedures to be adopted in carrying out a certain activity. The prime function of management is decision-making under such uncertain conditions, balancing the risk associated with a particular problem. A major accomplishment of PERT is the help it gives for managerial decision-making under uncertain conditions.

2. PERT system of three time estimates

2.1 The traditional single estimate of duration of any activity is replaced by three time estimates in PERT system-an optimistic time, a pessimistic time, and a most likely time.

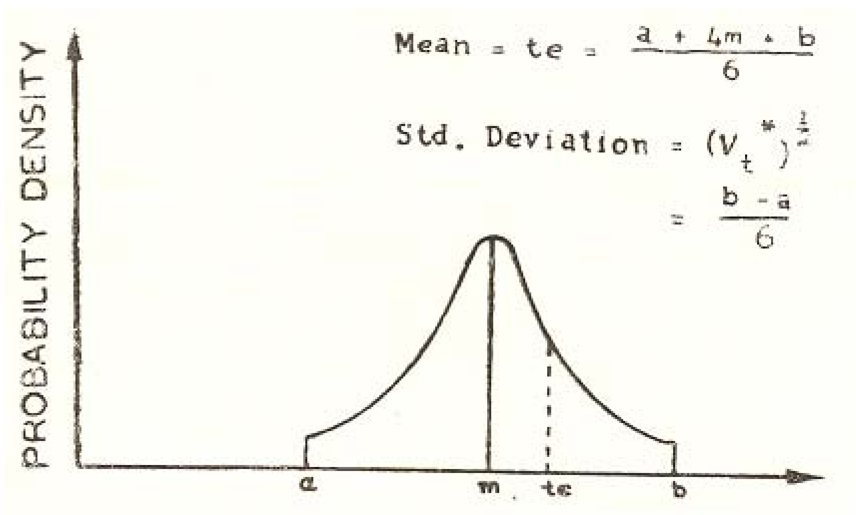
Optimistic time (a):-is that time estimate of an activity when everything is assumed to go well as per plan.

Pessimistic time (b):-the unlikely but possible performance time if whatever could go wrong, goes wrong in series.

Most likely time (m):-the time which the activity will take most frequently if performed a number of times-the model value.

2.2 This range of time provides a measure of uncertainty associated with the actual time required to perform the activity. On the basis of these estimates it is possible to derive the probability that a certain project will be completed before a specified date. The result is a meaningful and quite useful tool. A knowledge of this probability aspect together with an insight of the consequences of not meeting a scheduled date and the cost of expediting the project in various ways gives management a confidence in decision-making and instituting corrective actions. Once a management policy is established as to what constitutes a desirable level of risk in meeting schedules a schedule can be developed which is based on that policy and which uses the basic PERT network data. For each, activity an appropriate date would be determined having the same given level of risk.

2.3 In making PERT computations it should be understood that the distribution of activity performance time is purely hypothetical. In the absence of actual performance of similar projects statistical sampling is not possible. Hence the values. of a, m, b, rather depends on the judgement based on experience of the person ia charge of the activity in question. Considering this distribution of activity performance times to be approximately beta distribution, a unimodal distribution, the mean and the standard deviation would be as follows:



ERFORMANCE TIME=t *

Fig. 1 : Distribution of Activity Performance Time

3. Statistical treatment of time distribution

3.1 An important statistical machinery, i.e., Central Limit Theorem is needed for PERT; this theorem may be stated as follows:

3.1.1 Suppose m independent tasks are to be performed in order (may be the m activities which lie on the critical path of a network). Let t_1*,t.*, ... tm* be the times these tasks actually have taken for completion. Note that these are random variables with true means t_1, t_2…. t_m and true variances vt_1, vt_2*,…and these actual times are unknown till these specific tasks are actually performed.

Let $T^*=t_1+t_2^*+. .. +tm^*$

It should be noted that T· being a random variable also has a distribution. 3.1.2 The Central Limit Theorem states that if m is large the distribution of

T* is approximately normal with mean T and variance VT* given by·

$T=t_1+t_2, ... +tm$

$V^*_T=V^*_{t_1}+V^*_{t_2}+ ... V^*_{tm}$

That is, the mean of the sum, is the sum of the means; the variance of the sum is the urn of the variances; and the distribution of the sum of activity times will be normal regardless of the shape of the distribution of actual activity performance times.

3.2 The normal distribution is extensively tabulated and therefore probability statements can the made regarding the random variable T* by using these tables.

4. Illustration of PERT statistical approach

4.1 To illustrate the computation in the PERT statistical approach let us take the simple network of Fig. 2 with time values of a-m-b for each activity as estimated. For example a=l, m=2 and b=3 for activity 1-2. In the lower diagram of Fig. 2 the values of t4 and V~'" are indicated as computed by the following two equations :

$$\text{Mean} = t_e = \frac{a+4m+b}{6} \qquad \dots (1)$$

$$\text{Variance} = V_t^* = \left[\frac{b-a}{6} \right]^2 \qquad \dots (2)$$

sing the above two equations, for activity 1—2, the values are

$$t_e = \frac{1+4\times2+3}{6} = 2$$

$$V_t^* \left[\frac{(3-1)}{6} \right]^2 = \frac{1}{9}$$

4.2 After obtaining the values of the for each activity, the forward pass computations could be easily done to give the earliest expected event occurrence times Teas outlined in single time estimate system.

4.3 The event variance Vt* is then computed in a similar manner using the following rules:

(i) Vt* for the initial network event is assumed to be zero.

(ii) The Vt* for the event succeeding the activity in question is obtained by adding the activity's variance to the predecessor event's variance, except for merge events.

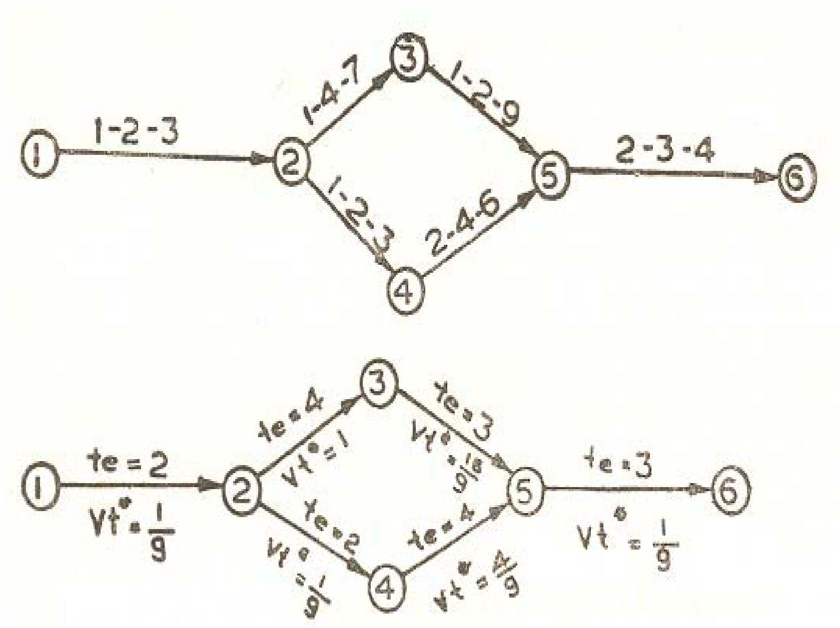


Fig. 2 : PERT statistical computations on a sample

(iii) At merge events VT* is computed along the same path used to obtain *TE;* i.e. the longest path. In case of ties, choose the path which gives larger variance.

4.4 Applying the above rules for calculation of Vt* for the network of Fig. 2, the following values are obtained.

V_T* for

Event 1 = 0

Event 2 = $0 + \frac{1}{9} = \frac{1}{9}$

Event 3 = $\frac{1}{9} + \frac{9}{9} = \frac{10}{9}$

Event 4 = $\frac{1}{9} + \frac{1}{9} = \frac{2}{9}$

Event 5 = $\frac{2}{9} + \frac{4}{9} = \frac{6}{9}$

or $\frac{10}{9} + \frac{16}{9} = \frac{26}{9}$ higher of the two values

Event 6 = $\frac{26}{9} + \frac{1}{9} = 3$

4.5 The backward pass and slack computations are also performed in the usually way.

5. Probability of meeting schedules

5.1 Scheduled dates are usually specified for significant events only marking an important stage in the project. Such events are usually called milestones that are likely to affect subsequent project activities vitally if not completed in time. The probability of occurrence of an event on or before a scheduled time is then an important information for management.

5.2 Referring to Fig. 2, the critical path is 1-2-3-5-6. Considering each of the activity duration on the critical path as random variables, the same isst1inptithl made during the process of collecting a. m and b values, the sum of these random variables T* also will be random variable governed by the Central limit theorem. Therefore,
and finally as per central limit theorem, the distribution of T* is approximately normal.

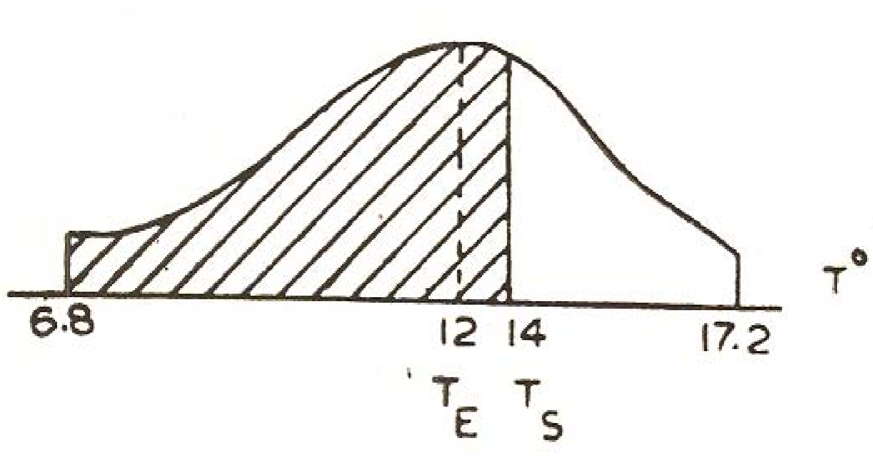


Fig. 3: Distribution of the Actual Occurrence Time T* of Event 6 (Fig. 2)

5.3 For computing probability of meeting a scheduled date of say 14, one has to really find the area which is shaded in Fig. 3. This probability can be read from the table of the normal curve areas once we find the value of *(Ts-TE)* i.e., (14-12) in terms of standard deviation (VT*)t

$$\text{Let } z = \frac{Ts-TE}{(V_T*)1/2}$$

Then In this example

$$Z = \frac{14-12}{(3)^{1/2}} = \frac{2}{1.73} = 1.16$$

5.4 A value of Z = 1.16 indicates that the scheduled time T s is 1.16 times standard deviation greater than the expected time *TE*= 12. Referring to table Page 83) for value of Z=1.l6, the corresponding probability value is 87.70 or approximately .88. This means that left to normal functioning without expediting the probability of completing the project within the schedule time of 14 is 88.

5.5 In some cases some of the interim events may have specific scheduled dates. In calculating the probability of meeting the scheduled date for the project and event it is assumed that all prior scheduled events occur on their scheduled dates. To compute such conditional probability of meeting a scheduled date, variances of the initial project event and all prior scheduled events should be set to zero and then usual variance and probability computations made .

Probability Aspects of Slack

1. Slack

1.1 The computed slack time is that which is expected, on the basis of schedules and estimates. Actual or observed slack will generally be different from the expected; its value will depend upon the exigencies of the activity and their probabilities. Theoretical analysis allows assumptions to be made about the actual slack that will develop.

1.2 In Fig. 1 on page 68, the expected slack is depicted in the top row. However, since change factors are at work, the slack may be smaller than anticipated; this is depicted in the middle row. In the same sense, actual slack may be larger than expected; chance may make the latest allowable time for an event fall earlier than the expected time. Such a situation is depicted in the bottom row.

2. Level of risk

2.1 Often a schedule will be determined for the completion of the objective event and/or other strategic events. The network computation described thus far provides a basis for determining the level of risk involved in meeting such a scheduled data Ts. PERT can be used to determine the risk involved in completing a given activity, in completing a portion of a programme or in completing an entire programme. These calculations permit adjustment of a schedule so as to arrive at a level of risk acceptable to management.

2.2 Risk is defined in terms of probability, which can be computed by the formula shown in Fig. 2. For the management to use this probability information, late performance must be evaluated, by cost penalties and other kinds of penalties for failure to meet obligations. The probabilities obtained provide only part of the information needed for management decisions.

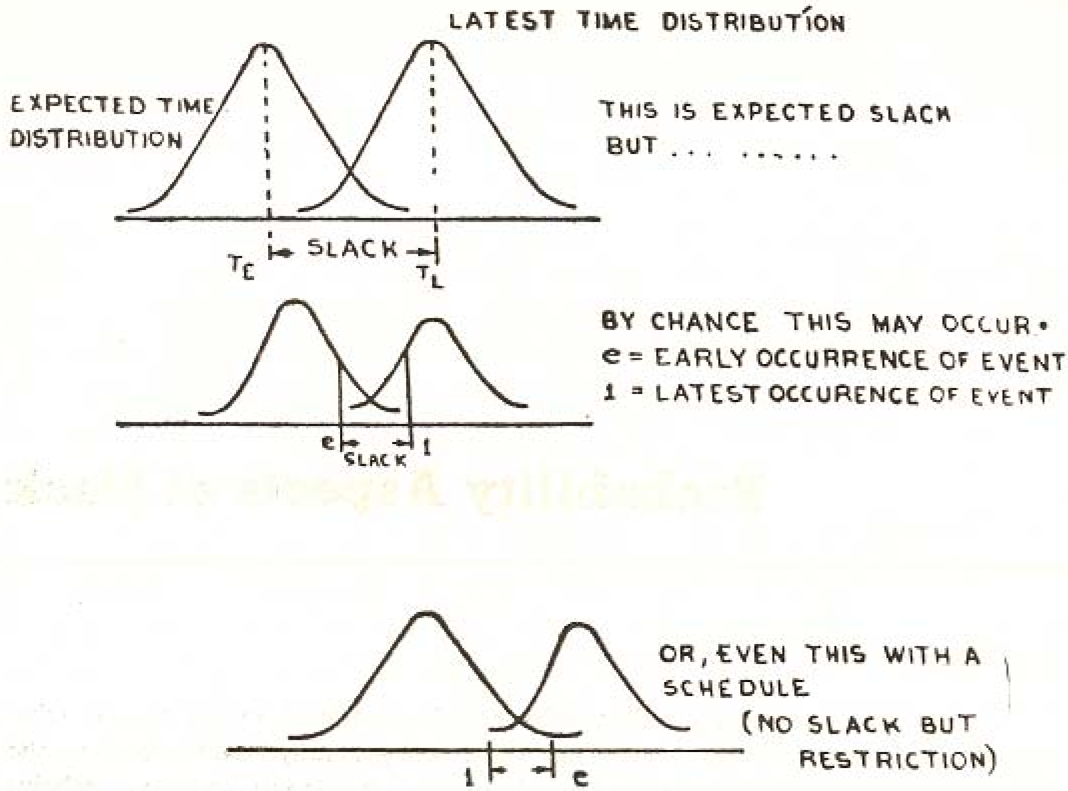


Fig. 1

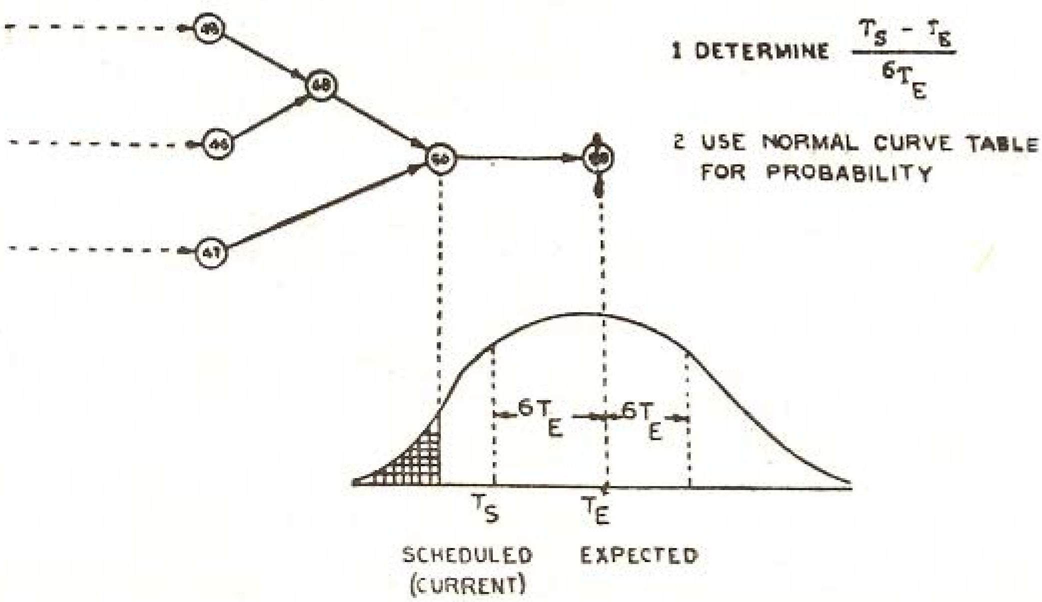


Fig. 2": Normal Distribution for a Scheduled Event

2.3 In the numerator of this formula, the expected time *TB* is subtracted from the scheduled time *Ts*. The quantity in the denominator is determined by summing the variances for the same activities used in arriving at *TE* and taking the square root of this sum. The result of these calculations may be referred to the normal curve probability table to determine the resulting level of probability.

2.4 An advantage of PERT is that this same class of calculations of probability can be used for positive management planning to establish dates as well as to adapt to given dates. Once a management policy is established as to what constitutes a desirable level of risk in meeting schedules, a schedule can be developed which is based on that policy and which uses the basis PERT network data. For each activity an appropriate date would be determined having the same given level of risk.

3. Probability of delay

3.1 The inclusion of probability of delay in determining the criticalness of events gives rise to another method of analyzing network problem areas which is complementary to the determination of slack. The probability of delay (which is the probability of zer90' negative slack) can be computed for the events, which are then ranked in order of criticalness. This method / yields no information about the amount of slack but does take into account the degree of certainty with which the original time estimates Were made.

3.2 A method of determining the probability of delay when positive slack exists is outlined below.

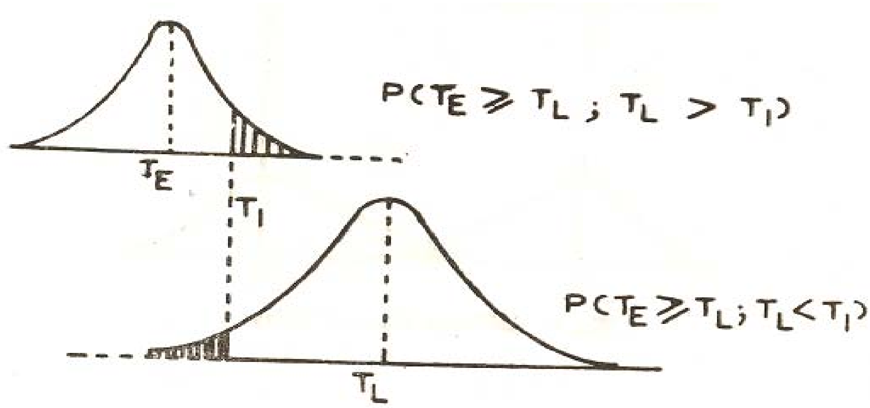


Fig. 3 : Probability of Delay

Assume a probability distribution for each *TE* and *TL;* these distributions represent the relative frequency of 'occurrence for various magnitudes of *TE* and *TL*. The point

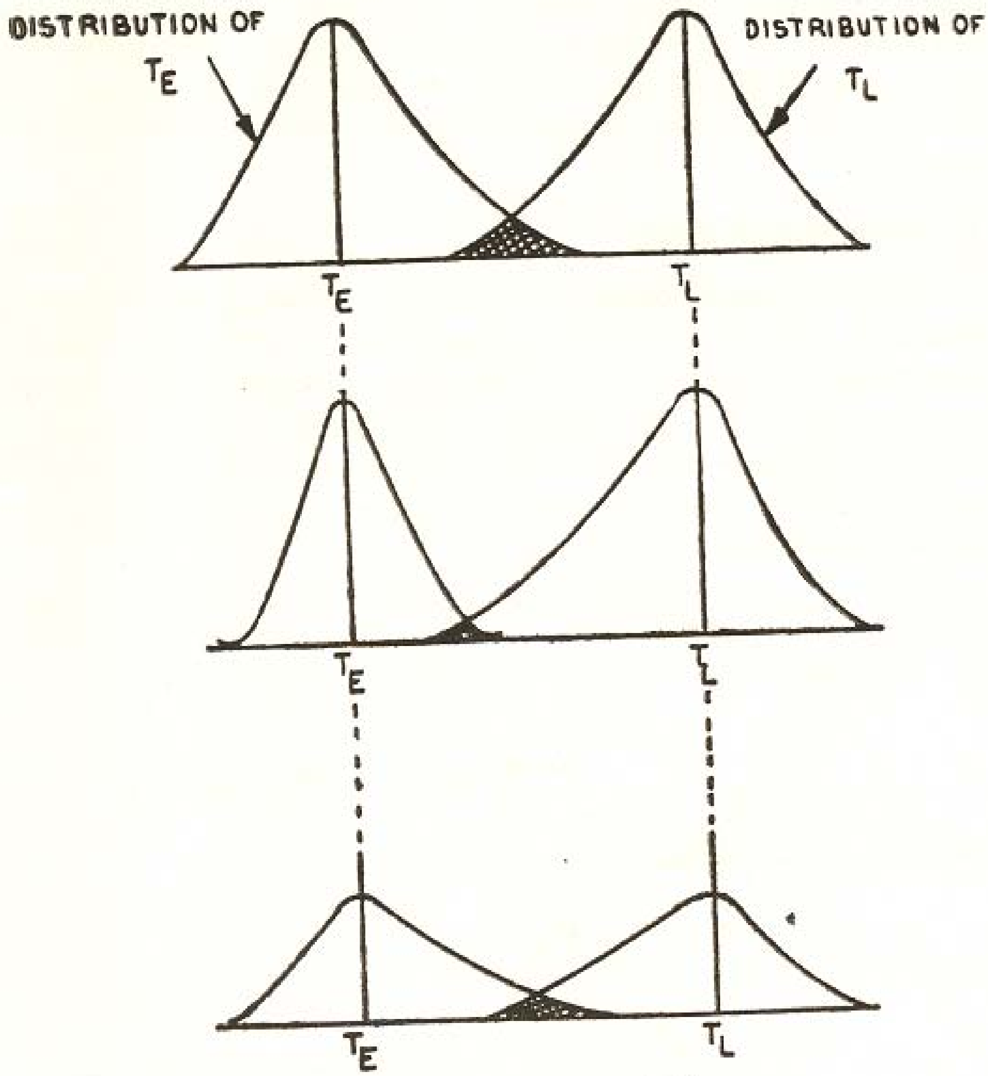


Fig. 4 : Different Values of Delay

in time where the distributions intersect is called T_1. Delay may occur *(T_E* greater than or equal to *T_L)* because of an extreme occurrence of *T_H*. The probability of this occurrence is shown in the illustration (Fig. 3) in the shaded area of the distribution .the *T_E* for those occasions where $T_L > T_I$. Similarly delay may occur because of an extreme occurrence of T_L.

3.3 The two distributions are independent, in a probability sense, because they represent the uncertainty in performance for two separate sets of activities those preceding and those following a given activity. Therefore, two probabilities can be summed up to determine the probability of delay for all occasions. This is achieved by finding TI, computing separately the probabilities for the two distributions and then adding together the two probability values obtained.

3.4 In the illustration of Fig. 4 are examples of different probabilities for delay values which may be obtained with. a given amount of slack. The contributions of the uncertainty associated with T_E add *T_L* is apparent by the difference in shaded areas. The analysis of slack and of probability of delay will often yield similar rankings for a set of events; however, there are occasions when an event, having appreciable positive slack, actually has a relatively high probability of no slack; and, for this reason, it is more likely to cause a delay than in comparable event with less slack.

EXERCISE
Indo-Easy Project

1 . Introduction

1.1 Kapur Ltd., of Bombay is an engineering firm collaborating with a foreign company for the local manufacture of an electrical washing machine of a modern design. The company is being advised by the collaborator on the preliminary programme to be undertaken before the machine will be put on the market.

1.2 The machine will be made of all-indigenous parts. It will be named "The Indo- Easy" washing machine.

1.3 The necessary activities have been detailed on the attached memorandum, and you have been requested to analyze the data and determine the critical activities. You are asked also to determine a reasonable project-time estimate.

2. Memorandum

The following are the details of a programme we believe will be necessary to plan for the introduction of the washing machine you have called "The Indo Easy".

2.1 Conduct a market research to determine the nature of a potential market for this product.

2.2 After this has been completed, develop price-and-demand schedules.

Also:

(i) Engineering Research is begun at this time.

(ii) Product Planning Specifications are to be developed now.

2.3 After Engineering Research is concluded, a laboratory model of the IndoEasy is to be made. (The model is influenced by the product-plan ding specification that have been laid out).

2.4 The next activity to be considered is "Conduct Product Appraisal" which follows "make laboratory model".

2.5 Before the product appraisal can be started, however, cost estimates must be prepared. (these follow the development of product-planning specifications)

2.6 At the same time as the product appraisal is begun, two other activities arc Also undertaken

(i) An anticipated profit-arid-loss statement is prepared; and

(ii) A patent search is Undertaken.

(Note that the profit-and loss analysis is affect a by the price and demand schedules prepared earlier).

2.7 When the product appraisal is completed, then

(i) The final product is designed, and

(ii) Sales force training is undertaken.

(Note that the final design is influence by the patent search results).

2.8 After the profit-and-Loss analysis bas ben completed the sales price is to be determined. (The price is influenced by the design of the final product).

2.9 Distribution channels are now developed (This requires the prior completion of sales force training, and of price determination for the product).

2.10 After the final product design is available, drawings and specifications arc to be prepared and issued. Also, advertising is to be prepared).

2.11 After drawings and specifications are available, manufacturing and related activities are undertaken. These include:

(i) Procure raw materials;

(ii) Establish manufacturing methods;

(iii) Procure confidants to tie purchased on the market (By Items');

(iv) Prepare service literature;

(v) Design and procure packaging of auxiliary items.

2.12 After manufacturing methods are established, production personnel are to be trained.

(Note that material procurement must be completed and production personnel trained before manufacturing can begin).

2.13 The items kapur Ltd., will manufacture ("Make Items") are now manufactured.

After manufacture of "Make Items" is complete, the components b ought earlier on the market ("Buy Items") must also be available before assembly can begin

2.14 The Indo-Easy is now to be assembled, and then tested.

2.15 After service literature is available, a service organization is to be trained. (This training must be completed, along with the testing of the machine, before the machine is "boxed, packed and shipped").

2.16 " Design and procure packaging" must be completed before boxing, packing and shipping can occur.

2.17 After the advertising is prepared, it is to be released nationally, and this also must be completed before the machine will be boxed, packed and shipped.

2.18 The distribution channels must be fully developed before the Indo-Easy may be boxed, packed and shipped.

2.19 The final activity is : "Box, Pack and Ship".

2.20 We hope the time to complete this progress may not exceed nine months, as our arrangements for collaboration are based on the assumption that Kapur Ltd. Canbeinproductionbythattime.

Problems for exercise on project "Indo-Easy"

1. Based on Memorandum develop the actual inter-relationships of the activities of the project.
2. Using the inter-relationships develop the project Network by visual method.
3. Convert the three time estimates into single time estimates and use them for carrying our 'Forward Pass' and 'Backward Pass' computations on the network. Identify the critical path.
4. Reproduce the network and Activity-oriented Matrix and calculate 'Slack' for each activity.
5. Reproduce the network on Event-oriented Matrix and calculate 'Slack' at each event.
6. Using the single time estimates prepare a 'Squared' network.

3. Data

ESTIMATED TIME FOR INDO-EASY PROJECT

	Activity	*Time in weeks*		
		a	*m*	*b*
A.	Conduct Market Research	2	3	6
B.	Develop price Demand Schedule	1	1½	3
C.	Conduct Engin. Research	3	5	8
D.	Develop Product Planning Specifications	1	2	4
E.	Develop Lab. Model	2	4	8
F.	Prepare Cost Estimates	2	3	4
G.	Conduct Patent Search	3	4	6
H.	Conduct Product Appraisal	2	2½	6
I.	Profit & Loss Analysis	1	1½	3
J.	Design Final Product	1	2½	4
K.	Train Sales Force	2	4	8
L.	Issue Drawings and Specification	2	3	6
M.	Prepare Advertising	5	5¾	11
N.	Determine Price	1	3	5
O.	Set Manufacturing Methods	2	4	8
P.	Procure Raw Material	4	6	10
Q.	Procure Buy Item	4	6	10
R.	Prepare Service Literature	2	4	10
S.	Design & Procure Packaging	3	5	8
T.	Release Advertising	2	4	6
U.	Develop Distribution Channel	3	4	6
V.	Train Service Organisation	1½	2	6
W.	Train Production Personnel	2	4	10
X.	Manufacture "Make Items"	1	3	8
Y.	Assemble	3	4	6
Z.	Test	4	5	8
I.	Box, Pack & Ship	1	1½	3

EXERCISE
Project Ray-See

1. Project History

1.1 In the manufacture of certain defence materials, the Defence Supply Agencies have reported difficulties experienced by manufacturers in determining the critical thickness and density of certain materials within specified tolerances.

1.2 It is especially desirable that certain critical items of defence material not be destroyed; and in some cases not even touched, when such measurements are made.

1.3 Also, it has been considered most important that a system capable of regulating the manufacture of these materials be provided to basic equipme9t manufacturers. Project RAY-SEE is to be undertaken to provide to all manufacturers on a priority basis a device that will fill the stated need.

2. Project Proposal

2.1 This new device utilises nuclear radiations. By using it, it will be possible to measure the thickness and density of sheets, plates, layers of powder and like materials without physical contact with the materials, and so without their destruction.

2.2 In the anticipated appliance, the material to be measured passes between the upper arm and lower arm. The head of the upper arm contains a radioactive S9.l1fce that sends a beam through the material. The lower arm provides means for determining the intensity of the beam, which has passed through the material.

2.3 A scale is calibrated beforehand by using the same source with various known thickness of the same material. This scale is used for reading the thickness of the layer.

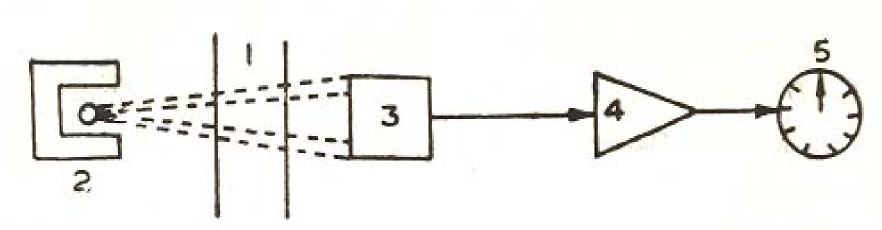


Fig. 1: Sketch of the Working of the Ray-See Device

1. Material to be measured; 2. Source and its holder; 3. Receiver; 4. Amplifier; 5. Meter.

2.4 Fig. 1 shows diagrammatically how the appliance works. The circuit, however. is more complicated than is shown and includes, an ionization chamber into which the residual beam passes. This residual beam gives rise to an electric current proportional to its intensity. The thickness of the material is then directly read from the recalibrated meter.

2.5 A great advantage of the appliance is that it measures without touching or destroying the material and can give continuous readings during the process of manufacture of fabrics, tubes, etc. It may also form part of a system regulating the manufacture of these materials.

The appliance intended for this purpose rises:

Alpha Rays for measuring paper, films. thin plastics etc.

Beta Rays for measuring thin sheet metal, plastic tubes, card-board etc.

Gamma Rays for measuring thick metal tubes and plates.

The source of those rays may be :

Alpha Rays: Uranium, radium, polonium.

Beta Rays: Ruthenium 106, strontium 90, Krypton 85, Cerium 144.

Gamma Rays: Cesium 137, Cobalt 60.

2.6 The provision of this appliance is to be undertaken on a priority basis and it ;s requested that project times and costs be determined on normal manufacturing procedures and on a crash basis to facilitate planning.

3. Data for Project Ray-See

3.1 The main parts for the device to be constructed are;

(i) Head

(ii) Arms

(iii) Ionization Chamber Components ({purchased)

(iv) Meter (Purchased)

(v) Radioactive Source of Alpha Rays: (Radium)
Radioactive Source of Beta Rays: (Strontium 90)
Radioactive Source of Gamma Rays: (Cobalt 60).

3.2 The procedures used and expected times are

Procedure		t_e (time units)
(i)	Design Entire Device	3
(ii)	Head:	
	(a) Draft	2
	(b) Make pattern	3
	(c) Cast	2
	(d) Machine	1
(iii)	Arms;	
	(a) Draft	2
	(b) Make pattern	3
	(c) Cast	2
	(d) Machine	1
(iv)	Meter:	
	(a) Purchase	2
	(b) Calibrate & T cst	2
(v)	Radium, Strontium 90 and Cobalt 60	
	(a) Purchase	6
(vi)	Ionization Chamber Parts:	
	(a) Purchase components	4
	(b) Assemble components	3

3.3 Assemble: (Figure in parentheses are time units t_e required)

Head:

After machining head, radium, strontium 90 or cobalt 60 may be as· sembled to head (2).

Arms:

After arms are machined, the assembled Ionization chamber is assembled to them (4).

Meter:

After sub-assembly of Radium etc., to Head (2), Meter is assembled to unit in final assembly along with arms sub-assembly (2).

As soon as meter is calibrated and tested, test-run plans may be drawn up (2).

After final assembly (2) test run of device is undertaken (2).

3.4 Data for Cost Analysis-Project RAY-SEE

	Activity		*Normal*		*Crash*
	Time		*Cost*	*Time*	*Cost*
A.	Design	3	150	3	
B.	Buy Radium	6	300	6	
C.	Buy I.C.	4	100	2	300
D.	Assemble I.C.	3	60	1	300
E.	Draft Arms	2	80	1	120
F.	Make Patts (arm)	3	60	1	260
G.	Cast Arm	2	40	11/2	60
H.	Machine Arms	1	40	1	100
I.	Assemble I.C. to arms	4	100	3	120
J.	Final Assem.	2	40	11/2	130
K.	Test Run	2	20	1	100
L.	Draft Head	2	80	1	120
M.	Make Path (Head)	3	60	1	260
N.	Cast Head	2	40	11/2	60
O.	Machine Head	1	40	1 T	100
P.	Sub Assem. R & H	2	40	1	120
Q.	Buy Meter	2	150	1	400
R.	Calibrate & Test Meter	2	20	2	
S.	Prepare for Test Run	2	70	1	150
	Total Normal Cost= 1490				

Overhead cost for varying time requirements=

Normal:	1900	16 units	1410	14 units	1200
18 units	1675	15½ units	1270	13½ units	1190
17 units	1520	15 units	1250	13 units	1180
				12½ units	1175

3.5 3 time values

		a	m	b
Where t_e is 1	3 times are	.5	.75	2.5
Where t_e is 2	3 times are	0.1	2.225	3.0
Where t_e is 3	3 times are	0.4	3.15	5.0
Where t_e is 4	3 times are	0.3	3.9	8.0
Where t_e is 6	3 times are	2.0	6.0	10.0

Problems for Exercise on Project Ray-See

1. Draw an interrelationship chart of all activities of the project Ray-See.
2. Prepare the network using these data and identify critical path.
3. Make a squared network f6r the project Ray-See.
4. Find out slacks for each activity by Activity-oriented Matrix method.
5. Find out slacks for each event by Event-oriented Matrix method.
6. Determine the probability of No-slack for each event and other pertinent data.
7. Use cost-data to determine optimum time and cost in a crash problem ignoring manpower leveling.

8. MANPOWER LEVELLING

Data for the project remaining the same, prepare a hew squared network to. meet the following specifications:

(i) Possibility and costs for expedited performance of activities remain as given before.

(ii) The company making the prototype of this item is small and willies to. avoid Having similar tasks performed in parallel, e.g. .. Draft bead and Draft Arms should not be scheduled for performance at the same time. Similarly cast arms and cast head, make patterns, and machining and assailable operations should follow each other to the expert possible, rather than overlap each other.

(iii) The designing department request that the purchase of the meter be scheduled as late as possible. It also requests that the purchase of the meter should not begin, if possible, until the radioactive materials and the ionization chamber are available. The management is willing to pay an extra premium up to 650 units of cost if this can be accomplished.

(iv) It is desired that there be a probability of at least 50 per cent that the test run be completed no later than 15 ul1its of time after the designs have been started. There is no particular advantage in having tbe prototype available before that time, as following sections of the programme are scheduled to begin only at this data. Management is interested in knowing what tbe expediting cost is likely to be to achieve this scheduled date.

9. Prepare a squared network of the final arrangement you propose to accomplish this goal.

10. SCHEDULE EV A LUATION AND ADJUSTMENTS.

(i) Assuming the square network finally to be as determined in 9 above, and that creation of the prototype in project RAY-SEE is underway, consider the action that would be taken under the followings conditions :

(ii) Assume the time unit is one week, and the 15 weeks are dated as follows:

Week	*Date*	*Week*	*Date*	*Week*	*Date*
1	Feb. 3—9	6	10—16	11	14—20
2	10—16	7	17—23	12	21—27
3	17—23	8	24—30	13	28—May 4
4	24—March 2	9	31—April 6	14	5—11
5	3—9	10	7—13	15	12—18

(Locate these under the squared network)

Problem

Assume we are now at the end of week 6 (March 16). The condition of the activities is as follows :

(i) Design work as completed on schedule.

(ii) . Procurement of radium and other radioactive material is considered on schedule.

(iii) Drafting of arms is completed but was finished 1/2 week behind schedule, due to revised specifications. New patterns are completed but starting of casting of arms is expected to be delayed also by one full week.

(iv) Drafting of Head will be finished 1/2 week behind schedule because of delay in drafting of arms, and now "make patterns for Head" is 1/2 week behind schedule.

(v) Ionization chamber components have just arrived! Week behind schedule.

Using the squared network as a guide, and referring to data for cost analysis in item 3.4 decide what action is to be taken at this time.

Table of Normal Distribution

Simplified Table

Range of Std. Dev. *a*	*Items falling within this range, % b*	*Items falling outside this range % b*
0.1	4.0	46.0
0.2	8.0	42.0
0.3	11.8	38.2
0.4	15.5	34.5
0.5	19.1	30.9
0.6	22.6	27.4
0.7	25.8	24.2
0.8	28.8	21.2
0.9	31.6	18.4
1.0	34.1	15.9
1.1	36.4	13.6
1.16	—	12.3
1.2	38.5	11.5
1.3	40.3	9.7
1.4	41.9	8.1
1.5	43.3	6.7
1.6	44.5	5.5
1.8	46.4	3.6
1.9	47.1	2.9
2.0	47.7	2.3
2.3	48.9	1.1
2.6	49.5	0.5
3.0	49.9	0.1

a. Range to either side of the distribution average.

b In place of the term "items" the terms units, articles, values, observations, or measurements may be substituted, depending upon the particular use of the table.

Detailed Table

Range of Std. Dev.	*Items within Std. Dev. range* %	*Range of Std. Dev.*	*Items within Std. Dev. Range* %	*Range of Std. Dev.*	*Items within Std. Dev. Range* %	*Range of Std. Dev.*	*Items within Std. Dev. Range* %
0.01	0.399	0.36	14.058	0.71	26.1I 5	1.06	35.543
0.02	0.798	0.37	14.431	0.72	26.424	1.07	35.769
0.03	1.197	0.38	14.803	0.73	26.730	1.08	35.993
0.04	1.595	0.39	15.173	0.74	27.035	1.09	35.214
0.05	1.994	0.40	15.542	0.75	27.337	1.10	36.433
0.06	2.392	0.41	15.910	0.76	27.637	1.11	36.650
0.07	2.790	0.42	16.276	0.77	27.935	1.12	36.864
0.08	3.188	0.43	16.640	0.78	28.230	1.13	37.076
0.09	3.586	0.44	17.003	0.79	28.524	1.14	37.286
0.10	3.983	0.45	17.364	0.80	21.814	1.15	37.493
0.11	4.380	0.46	17.724	0.8i	29.103	1.16	37.698
0.12	4.776	0.47	18.082	0.82	29.389	1.17	37.900
0.13	5.172	0.48	18439	0.83	29.673	1.18	38.100
0.14	5.567	0.49	18.793	0.84	29.955	1.19	38.298
0.15	5.962	0.50	19.1-l6	0.85	30.234	1.20	38.493
0.16	6.356	0.51	19.497	0.86	30.511	1.21	38.686
0.17	6.749	0.52	19.847	0.87	30.785	1.22	38.877
0.18	7.142	0.53	20.194	0.88	31.057	1.23	39.065
0.19	7.535	0.54	20.540	0.89	31.327	1.24	39.251
.0.20	7.926	0.55	20.884	0.90	31.594	1.25	39.435
0.21	8.3.17	0.56	21.226	0.91	31.859	1.26	39.617
0.22	8.706	0.57	21.566	0.92	32.121	1.27	39.796
0.23	.9.095	0.58	21.904	0.93	32.281	1.28	39.973
0.24	9.483	0.59	22.240	0.9-l	32.639	1.29	40.147
0.25	9.871	0.GO	22.575	0.95	32.849	1.30	40.320
0.26	10.257	0.61	22.907	0.96	33.147	1.31	40.490
0.27	10,642	0.62	23.237	0.97	33.398	1.32	40.658
0.28	11.026	0.63	23.565	0.98	33.646	1.33	40.824
0.29	11.409	0.64	23.891	0.99	33.891	1.34	40.988
0.30	11.791	0.65	24.215	1.00	34.134	1.35	41.149
0.31	12.172	0.66	24.537	1.01	34.375	1.36	41.309
0.32	12552	0.67	24.857	1.02	34.614	1.37	41.466
0.33	12.930	0.68	25.175	1.03	34.850	1.38	41.621
0.34	13.307	069	25.490	1.04	35.083	1.39	41.774
0.35	13.683	0.70	25.804	1.05	35.314	1.40	41.924

(Contd.)

Range of Std Dev.	*Items within Std. Dev. range,* %	*Range of Std Dev.*	*Items within Std. Dev. range,* %	*Range of Std. Dev.*	*Items within Std Dev. range,* %	*Range of Std. Dev.*	*Items within Std. Dev range,* %
1.41	42.073	1.76	46.080	2.11	48.257	2.46	49.305
1.42	42.220	1.77	46.164	2.12	48.300	2.47	49.324
1.43	42.364	1.78	46.246	2.13	48.341	2.48	49.343
1.44	42.507	1.79	46.327	2.14	48.382	2.49	49.361
1.45	42.647	1.80	46.407	2.15	48.422	2.50	49.379
1.46	42.786	1.81	46.485	2.16	48.461	2.51	49.396
1.47	42.922	1.82	46.562	2.17	48.500	2.52	49.413
1.48	43.056	1.83	46.638	2.18	48.537	2.53	49.430
1.49	43.189	1.84	46.712	2.19	48.574	2.54	49.446
1.50	43.319	1.85	46.784	2.20	48.610	2.55	49.461
1.51	43.448	1.86	46.856	2.21	48.645	3.56	49.477
1.52	43.574	1.87	46.926	2.22	48.679	2.57	49.492
1.53	43.699	1.88	46.995	2.23	48.713	2.58	49.506
1.54	43.822	1.89	47.062	2.24	48.745	2.59	49.520
1.55	43.943	1.90	47.128	2.25	48.778	2.60	49.534
1.56	44.062	1.91	47.193	2.26	48.809	2.61	49.547
1.57	44.179	1.92	47.257	2.27	48.840	2.62	49.560
1.58	44.295	1.93	47.320	2.28	48.870	2.63	49.573
1.59	44.408	1.94	47.381	2.29	48.899	2.64	49.585
1.60	44.520	1.95	47.441	2.30	48.928	2.65	49.598
1.61	44.630	1.96	47.500	2.31	48.956	2.66	49.609
1.62	44.738	1.97	47.558	2.32	48.983	2.67	49.621
1.63	44.845	1.98	47.615	2.33	49.010	2.68	49.632
1.64	44.950	1.99	47.670	2.34	49.036	2.69	49.643
1.65	45.053	2.00	47.725	2.35	49.061	2.70	49.653
1.66	45.154	2.01	47.778	2.36	49.086	2.71	49.664
1.67	45.254	2.02	47.831	2.37	49.111	2.72	49.674
1.68	45.352	2.03	47.882	2.38	49.134	2.73	49.683
1.69	45.449	2.04	47.932	2.39	49.158	2.74	49.693
1.70	45.543	2.05	47.982	2.40	49.180	2.75	49.702
1.71	45.637	2.06	48.030	2.41	49.202	2.76	49.711
1.72	45.728	2.07	48.077	2.42	49.224	2.77	49.720
1.73	45.818	2.08	48.124	2.43	49.245	2.78	49.728
1.74	45.907	2.09	48.169	2.44	49.266	2.79	49.736
1.75	45.994	2.10	48.214	2.45	49.286	2.80	49.744

(Contd.)

Range of Std. Dev.	*Items within Std. Dev. range,* %	*Range of Std. Dev.*	*Items within Std. Dev. range,* %	*Range of Std. Dev.*	*Items within Std. Dev. range,* %	*Range of Std. Dev.*	*Items within Std. Dev. range,* %
2.81	49.752	3.11	49.906	3.41	49.968	3.71	49.990
2.82	49.760	3.12	49.910	3.42	49.969	3.72	49.990
2.83	49.767	3.13	49.913	3043	49.970	3.73	49.990
2.84	49.774	3.14	49.916	3.44	49.971	3.74	49.991
2.85	49.781	3.15	49.918	3.45	49.972	3.75	49.991
2.86	49.788	3.16	49.921	3.46	49.973	3.76	49.992
2.87	49.795	3.17	49.924	3.47	49.974	3.77	49.992
2.88	49.801	3.18	49.926	3.48	49.975	3.78	49.992
2.89	49.807	3.19	49.929	3.49	49.976	3.79	49.992
2.90	49.81J	3.20	49.931	3.50	49.977	3.80	49.993
2.91	49.819	3.21	42.934	3.51	49.978	3.81	49.993
2.92	49.825	3.22	49.936	3.52	49.978	3.82	49.993
2.93	49.831	3.23	49.938	3.53	49.979	3.83	49.994
2.94	49.836	3.24	49.940	3.54	49.980	3.84	49.994
2.95	49.841	3.25	49.942	3.55	49.981	3.85	49.994
2.96	49.846	3.26	49.944	3.56	49.981	3.86	49.994
2.97	49.851	3.27	49.946	3.57	49.982	3.87	49.995
2.98	49.856	3.28	49.948	3.58	49.983	3.88	49.995
2.99	49.861	3.29	49.950	3.59	49.983	3.89	49.995
3.00	49.865	3.30	49.952	3.60	49.984	3.90	49.995
3.01	49.869	3.31	49.953	3.61	49.985	3.91	49.995
3.02	49.874	3.32	49.955	3.62	49.985	3.92	49.996
3.03	49.878	3.33	49.957	3.63	49.986	3.93	49.996
3.04	49.882	3.34	49.958	3.64	49.986	3.94	49.996
3.05	49.886	3.35	49.960	3.65	49.<;87	3.95	49.996
3.06	49.889	3.36	49.961	3.66	49.987	3.96	49.996
3.07	49.893	3.37	49.962	3.67	49.988	3.97	49.996
3.08	49.897	3.38	49.964	3.68	49.988	3.98	49.997
3.09	49.900	3.39	49.965	3.69	49.989	3.99	49.997
3.10	49.903	3.40	49.966	3.70	49.989	4.00	49.998